STRENGTH OF LONG LINES IN TERRESTRIAL GEODETIC CONTROL NETWORKS

Abstract

A knowledge of the a posteriori accuracy of long lines in classical triangulation networks is important not only for establishing the quality of existing primary national control networks, but also for assessing the likely contribution of satellite derived observations. Extensive tests carried out with various networks, ranging from a basic triangulation chain to Block VI of the European Triangulation Network, show definite trends. The a posteriori standard errors of both the scale and the orientation of an adjusted line diminish considerably as the length of the line increases. In the case of medium sized countries, the results compare favourably with those predicted for satellite derived data. The conclusions emphasize the need for great care in the reduction of terrestrial observations and the elimination of all possible systematic errors before proceeding with a least squares adjustment.

1. Optimisation of Terrestrial Triangulation Networks

The strength of a network or its ability to propagate scale and orientation from one part of the network to another without undue deterioration, is based upon the accuracy of its adjusted angles (or side ratios). These, in turn, are affected by the shape of the network as well as by the density of the various types of observed quantities (angle, length, azimuth or position) and their respective variances and covariances. It is not too difficult to realize that measured distance ratios or observed azimuth differences constitute indirect measurements of angles, in addition to providing the network with direct determinations of scale and orientation respectively.

Simulation tests with basic triangulation networks (Ashkenazi and Dodson, 1974; Sykes, 1975) have shown that, when the a priori standard errors of the observed theodolite directions are almost equivalent (e.g. 1 sexagesimal second—of—arc and 5 parts—per—million) or lower than those of the measured distances, then the strength of the shape of the network, as expressed by the average a posteriori standard error of an angle, \( \sigma_a \), is entirely dependent upon the accuracy of the observed angles and is not affected by the number of measured distances. Observed Laplace azimuths and measured distances have no contribution to make other than lowering the values of \( \sigma_\beta \) and \( \sigma_\gamma \), the average a posteriori standard errors of orientation and scale respectively.

This conclusion has important implications in the case of large terrestrial national and continental triangulation networks. Many of the well established national
control networks, particularly in Europe, tend to be dense triangulation schemes with a variable number of Laplace azimuths and electro-magnetically measured distances, the latter with a priori standard errors comparable to those of the observed angles. For example, in the case of Block VI of the European Triangulation (consisting of the combined primary control networks of the United Kingdom and the Republic of Ireland), the average a priori standard errors for observed theodolite directions and measured distances are approximately 0.65 sec and 3 p.p.m. respectively.

Simulation tests with Block VI (Ashkenazi and Cross, 1971) confirmed earlier speculation that, in such dense triangulation networks, there is no need to measure all the side lengths or a large number of Laplace azimuths. In the case of Block VI, it was found that, beyond 200 measured distances and around 30 to 40 observed Laplace azimuths, the average a posteriori standard errors in scale and in orientation, \( \sigma_\alpha \) and \( \sigma_\beta \), could not be significantly reduced any further.

2. Short and Long Lines in Terrestrial Networks

Throughout these earlier network optimisation tests, the invariant criteria used to assess the strength of a network in scale and in orientation were the (average a posteriori) values of \( \sigma_\alpha \) and \( \sigma_\beta \) for a sample of short lines joining adjacent stations in the network. This decision seemed reasonable at the time (1969–70) considering that the main application of such primary control networks is the provision of scale and orientation control for lower order densification networks.

However, with the development of geodetic satellite methods and particularly satellite–Doppler techniques, it now appears that the strength of the scale and orientation of a terrestrial network along medium or long lines is just as important as along short lines. One would need to know the strength of a sample of long lines in an existing terrestrial triangulation network whenever one is contemplating a new mixed terrestrial–satellite solution. Only then it would be possible to evaluate the possible contribution of, say, satellite–Doppler derived position observations upon the current strength of the terrestrial network.

Conversely, one may use a long terrestrial triangulation chain to provide scale for a pure satellite–triangulation network, or orientation for a pure satellite–trilateration network. Lastly, very long terrestrial base–lines could also be used for a variety of geophysical or astronomical purposes. Clearly, in all these cases, a knowledge of the a posteriori accuracy of long lines, in existing or planned terrestrial networks, would be a distinct advantage.

Moreover, one could now raise the network optimisation problem in this context and ask oneself if the earlier conclusions arrived at by adopting the strength of short lines as optimisation criteria would differ from those based upon the strength of medium or long lines.

3. Test Networks: Block VI and Kenya

The problems raised in § 2 above were investigated in a new series of simulation tests, leading to this paper. Computer simulation of geodetic networks is now well established and amply documented (Ashkenazi, 1975). This particular series of tests was carried out by a newly developed “plane rectangular model” technique (Cross and Ashkenazi, 1975) which is used in conjunction with the plotting table of a photogrammetric plotter.