Mathematical Modelling of Marine Systems

Numerical studies on alluvia settlement, considering hydrodynamic interaction*

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Abstract — The paper addresses the model problem on the motion of alluvia clouds in an infinite fluid. Deposition fractions are described in close relationship with one another. For this type of calculation, only the finiteness of particles dimensions is taken into consideration. The motion of alluvia has been found to be closely dependent on the ratio between the size of granules. Time dependence of separation periods of fractions on the dimensionless radius for different densities of samples has been studied. It has been established that for particles of similar dimensions, calculation of the rate of sedimentation using Stokes’s formula is not possible.

INTRODUCTION

By resolving the problems of the sinking of particles in the shelf area, of the transport of sediments, gases, zoo- and phytoplankton, we simultaneously address diverse environmental issues.

Investigating the transport of alluvia, we have to deal with the following physical problems [1]:

(1) the determination of the velocity of movement and the size of settling particles;
(2) the improved description of the mechanism for the transport of suspended particles;
(3) description of the feature with secondary circulation superposing over the major flow.

Today, only conceptual models of sedimentation are available. The sedimentation process in the shelf area is fully described by heuristic models. Laboratory studies of sedimentation are normally conducted using sedimentometers, which hamper the mathematical/physical interpretation of the phenomena at issue. Furthermore, the notion of a fall is then introduced; this has the dimensionality of velocity, but it does not have any physical, or any mathematical sense of the velocity of the sinking particles. The formula for that notion reads:

\[ w_i = \frac{(L_1 - L_2)}{(t_1 - t_2)}, \]

where \( L_1 \) and \( L_2 \) are the heights of the passage of the \( i \)th fraction in two different cans; \( t_1 \) and \( t_2 \) indicate the time of the particles sedimentation in the cans, that is, the sinking velocity is the ratio between the difference of the heights of the two cans and the difference of the time of passage of the heights' suspension by the particles.

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The diagram for the dependence of the fall on the extent of the area crossed by the sample has an intricate wave shape and, apparently, depends on the configuration of the walls. Nonetheless, that notion is being widely employed today to study the movement of sediments.

Therefore we suggest to apply a different approach to the investigation of alluvial transport. It is well known that it is impossible or nearly impossible to conceive an axiomatizable mechanics for finite size particles. Therefore, for convenience, modern mechanics assumes the particles of matter to be infinitely small. The problem arises when we have to introduce some scale for examining the small but finite radius of particles. It is that scale that researchers adopt as a basic quantity in this type of investigation. Here, the latter quantity is represented by the compactness of packing \( \alpha \): \( \alpha = a/l \). Differentiating the size is accomplished by means of a dimensionless radius \( \beta \): \( \beta = a_1/a_1 \). As a standard radius, we use the fixed radius of model sample particles of the 1st grate.

The contemporary statements of the problem [1] suggest the following hypotheses:
1. The shape and the size of sediments are identical.
2. The friction layer generated by the particles (i.e. an insignificant turbulization of the fluid by the sinking sediments) is flat.
3. The flux is steady and homogeneous.

The rate of sedimentation is calculated using Stokes's formula [1–3]:

\[
V = \frac{2a^2g}{9\mu} (\rho_p - \rho_w).
\]

In this paper, the size and shape of sinking particle are assumed to be diverse; however, the shape, for simplicity, is taken to be spherical. The effect of the flux is not examined here. It will be discussed in another paper.

We have used in the study the numerical method of modelling with involvement of particles [4], as well as the hydrodynamic method of point forces developed by Struminsky et al. [5–10].

In refs 5–10, the researchers experimented with clouds of identical sinking particles. They have demonstrated the general interaction between spherical particles having densities larger and smaller that of the surrounding fluid (solid particles and air bubbles, respectively). Those works had been preceded by the researchers conducted in refs 11–13, where it had been established that the rate of sinking of particles in a cloud depends on the number of particles therein. For the first time, the issue of the relationship between the parameters of the sinking alluvial cloud and the individual size of particles was raised in ref. 11.

In ref. 5, it was suggested to use Lamb's analytical study on velocity potential [14], which was applied in ref. 15 to handle similar problems on multiparticle ensembles. Moreover, ref. 5 specified the limits of error resultant from the utilization of the respective approximations. Regrettably, in the Soviet hydrodynamics, only Struminsky's school emphasized the significance of the interaction of particles in a fluid and of the gradual transformation of small ensembles into large ones. In that context, they conducted the possible parameterization of the averaged characteristics, such as the effective viscosity and concentration [7].

Direct application of Batchelor's theory appears to be problematic, as it does not appear possible to directly parameterize statistical ensembles and to find the limiting case when the individual interaction of particles and the averaged parameter converge.