FIVE-DIMENSIONAL, NONSTATIONARY COSMOLOGICAL SOLUTIONS WITH ROTATION

E. V. Kuvshinova and V. F. Panov

Two nonstationary cosmological solutions of the five-dimensional Einstein equations are found for different metrics. In one case the sources of the gravitational field are an anisotropic fluid and a radiation field, while in the other case they are an anisotropic fluid, a radiation field, and a heat flux.

The number of papers on multidimensional cosmology has increased in recent years ([1-3] and others), and there has been continuing interest in the investigation of cosmology with rotation ([4-6] and others). Multidimensional cosmological models with rotation were proposed in [7, 8]. Metrics that generalize a four-dimensional metric of the Gödel type were used.

In the present paper, in which the results of [9] are developed, a five-dimensional, nonstationary cosmological solution with rotation is obtained for a metric in the form

$$dS^2 = dt^2 - R^2(t) [dx^2 + 2\kappa e^{m_x} dy^2 + dz^2] - 2R(t) e^{m_x} dy dt - t^4 dx^2,$$

where $R = vt$, $v$, $m$, and $k$ are constants, $v > 0$.

We have $t = x^0$, $x = x^1$, $y = x^2$, and $z = x^3$.

We seek a solution of the five-dimensional Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}$$

for the metric (1), assuming that the sources of the gravitational field in the given cosmological model are an anisotropic fluid and a radiation field.

The energy–momentum tensor (EMT) of the anisotropic fluid is

$$T_\mu^\nu = (\rho + \pi) u_\mu u^\nu + (\sigma - \pi) \chi_\mu \chi^\nu - \pi g_\mu^\nu,$$

where $u_\mu u^\mu = 1$, $\chi_\mu \chi^\mu = -1$, $\chi^\mu u_\mu = 0$, $\rho > 0$, and $\sigma > \pi$.

We take

$$u^\mu = \delta_0^\mu, \chi^\mu = \delta_1^\mu, \mu = 0, 1, 2, 3, 4.$$  

The nonzero components of the EMT of the anisotropic fluid then have the form

$$T_\nu^0 = \rho, T_\nu^2 = -\rho Re^{m_x}, T_1^1 = \pi R^2, T_1^2 = (\rho + \pi) R^2 e^{m_x} + \pi R^2 e^{m_x}, T_2^2 = \pi R^2.$$
The EMT of the radiation field is

\[ T_{\mu\nu} = \mathcal{W}_\mu \kappa_\nu, \]

where we have

\[ \kappa_\mu \kappa^\mu = 0. \]

We set \( \kappa_\mu = (\kappa_0, 0, \kappa_2, 0, 0) \) and \( \kappa_2 = \kappa_2 \mathbf{R}_\mathbf{e}^{\mu\nu}. \)

From (6) we can then assume

\[ \kappa_2 = \kappa_0 \{-1 - \sqrt{(\kappa + 1)}\}. \]

The nonzero components of the EMT of the radiation field are

\[ T'_{00} = \mathcal{W}_0^2, \quad T'_{02} = \mathcal{W}_0 \kappa_2, \quad T'_{22} = \mathcal{W}_2^2. \]

The EMT \( T_{\mu\nu} \) of our model is the sum of the EMT of the anisotropic fluid and the EMT of the radiation field. Writing out Eqs. (2) for the metric (1), with allowance for \( R = \nu t \), and assuming the functions to depend on different powers of \( t \), we obtain

\[ \pi = \frac{(-12 \nu^2 \kappa + 4 \kappa m^2 + 3 m^2)}{4 (\kappa + 1) \nu^2} t^{-2}, \quad \sigma = \frac{(-12 \nu^3 \kappa + m^4)}{4 (\kappa + 1) \nu^2} t^{-2}, \]

\[ \mathcal{W}_\mu \kappa_\nu^3 = \frac{6}{t^2 \sqrt{\nu + 1}}, \quad m^2 = \frac{-12 (1 + \sqrt{\kappa + 1}) \nu^2}{(\kappa + 1/2)}, \]

\[ \rho = -3 (\kappa + 2) (\kappa + 1/2) + 3 (4 \kappa + 1) (1 + \sqrt{\kappa + 1}) - 6 (\kappa + 1/2) \sqrt{\kappa + 1}. \]

The entire solution is expressed in terms of the parameters \( \kappa \) and \( \nu \); we set \( \kappa_0 = 1 \). To satisfy the inequalities \( m^2 > 0 \) and \( \sigma > \pi \), we impose the following restriction on \( \kappa \):

\[ -1 < \kappa < -1/2. \]

The inequality \( \rho > 0 \) can be satisfied for \( \kappa \) less than but fairly close to \(-1/2 \) [for \( \kappa = -1/2 - 10^{-5} \), for example].

Let us calculate the model’s kinematic parameters. Expansion of the model \( \theta = \nu^{t/\kappa} \), five-acceleration vector \( a_\mu = \nu_{\mu\nu} \nu^{1/\kappa} \), angular rotation rate tensor \( w_{\mu\nu} = -(1/2)(u_{\mu \nu} - u_{\nu \mu}) + (1/2)(a_{\mu \nu} - a_{\nu \mu}) \), displacement tensor \( \sigma_{\mu\nu} = -u_{\mu\nu} + a_{\mu\nu} - (1/3)\delta(u_{\mu \nu} - g_{\mu \nu}) \), rotation \( w = [(1/2)w_{\mu \nu} w^{\mu \nu}]^{1/2} \), acceleration \( a = (-a_{\mu} a^{\mu})^{1/2} \), displacement \( a = [1/2a_{\mu\nu} a^{\mu\nu}]^{1/2} \). For our model with the metric (1) we have

\[ \theta = t^{-1}, \quad a = \frac{1}{t \sqrt{\nu + 1}}, \quad w = \frac{|m|}{2 \nu \sqrt{\nu + 1} t}, \]

and \( \sigma = (61)^{1/2}(18)^{-1/2}t^{-1} \).

We also found another solution of the five-dimensional Einstein equations (2) for the metric

\[ dS^2 = dt^2 - R^2(t) dx^2 - R^2(t) dy^2 - R^2(t) dz^2 - ae^m z dx^3 - 2 ae^n dx^4 dt, \]

where \( R = \nu t; \quad V, \quad m, \quad a \) are constants, \( V > 0 \).

The sources of gravitation in the second model are an anisotropic fluid and a radiation field, and a heat flux is also taken into account.