QUANTUM MECHANICS IN THE $K$-FIELD FORMALISM: ACCOUNT OF INTERACTIONS WITH MAGNETIC FIELD

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Using the $K$-field formalism, quantum equations are derived for a particle in a random stationary electromagnetic field. It is shown that this field can enter the $K$-field expression only via the $|v(x^+)|^2$ functions (the squared classical velocity of the $K$-particle).

1. The fundamental principles of quantum mechanics in the $K$-field formalism have been reported in [1]. Using the de Broglie postulate in its geometrical formalization on the $kV^4$ manifold, the $K$-field equation was constructed, which makes it possible to estimate the $K$-field potential, $k$, for any quantum system. In [1], Lyapunov-stable solutions to the equations for the $K$-motions were shown to be mandatory for the development of a quantum-mechanics model of steady states in a quantum system satisfying the solutions to the $K$-field equation.

The purpose of this paper is to devise a logically consistent scheme for the magnetic field to be entered into the $K$-field formalism.

2. The mechanism already proposed in [1] was the introduction of the magnetic field into the $K$-field equation through a conventional scheme of gauge fields

$$[\nabla - (c/v)^2 n_0 A \partial_0]^2 k_0 = [(c/v)^2 \partial_0]^2 k_0,$$

where $v^2 = (k)^2 \theta_{00}$ is the squared particle velocity in the external electromagnetic field, $n_0 = e/cp^0$, and $p^0$ is the zero component of the 4-momentum.

Notwithstanding the simplicity and formal consistency of this method in considering interactions with the magnetic field (the mathematical formalism of the $K$-field theory provides for introduction of any additional fibres on the $kV^4$ manifold), attempts to apply it in practice have ended in failure, since no Lyapunov-stable solution to the $K$-motion equation satisfying the solutions of $K$-field equation (1) has been arrived at.

3. Let us consider a microparticle in an external stationary electromagnetic field. We assume that the steady state does exist for the quantum system in question. Then, according to the philosophy of the $K$-field formalism, the equation for the $K$-motions [1]

$$^{(k)}Vp^i = mg^{\mu \nu} (^{(k)}V_{[\mu} k_{\nu]}) dx^\nu/c,$$

where $v^2 = (k)^2 \theta_{00}$ is the squared particle velocity in the external electromagnetic field, $n_0 = e/cp^0$, and $p^0$ is the zero component of the 4-momentum.

From the definition of the absolute differential $^{(k)}V$ on the $kV^4$ manifold introduced in [1]

$$^{(k)}V_{i} k_0 = D_{i} k_0,$$

$$^{(k)}V_{i} k_0 = D_{i} k_0 -^{(k)}S_{lm} k_m,$$

where, according to [1], the quantity $^{(k)}S_{lm} k_m$ is related to the magnetic field by the following condition:

$$^{(k)}S_{lm} k_m dx^l = (e/cp^0) F_{lm} dx^0 k_m.$$
On these grounds, the equation for the $K$-motions (3) acquires the form

\begin{equation}
(k)\nabla p_i = mg^\nu D_i \left( k_\nu dx^\nu \right)/c - \left( m/c \right) \left( c/e \right) F^m_i dx^0 k_m .
\end{equation}

(7)

Given the fact that the $K$-field is in the $K$-frame $\partial_\mu k_\nu = \partial_\nu k_\mu$, the following equality is readily derived:

\begin{equation}
g^\nu D_i \left( k_\nu dx^\nu \right) = g^\nu D_k_i ,
\end{equation}

(8)

which makes it possible to rewrite the equation for the $K$-motions (7) in the following fashion:

\begin{equation}
(k)\nabla u_i / dt = \left( g^\nu D_k_i / dt \right)/c - \left( e/c \right) \left( g^\nu F^m_i k_m \right) ,
\end{equation}

(9)

provided that $u_i = p_i/m$.

On the other hand, according to the definition of the absolute differential $(k)\nabla$ on the $k V_4$ manifold (see [1])

\begin{equation}
(k)\nabla u_i / dt = du_i / dt - \left( e/p^0 \right) F^\mu_i u^\mu .
\end{equation}

(10)

Now, for the spatial component of the 4-velocity, $u_i$, of the test particle, which is the classical model of a microparticle, we obtain

\begin{equation}
du_i / dt = \left( g^\nu D_k_i / c \right) + \left( e/m \right) F^i_j \left[ u^j - g^j i k_i/c \right] / p^0 .
\end{equation}

(11)

If we introduce the notation

\begin{equation}
(0) u_i = u_i - g^i i k_i/c ,
\end{equation}

(12)

equation (11) will acquire the form of the classical Lorentz equation for a point charge in the electromagnetic field [2]

\begin{equation}
d(0) u_i / dt = \left( e/p^0 \right) F^\mu_i (0) u^\mu .
\end{equation}

(13)

Summing up the above, we conclude that to construct the $K$-motion equation for the $K$-particle (a point particle representing the classical model of the quantum system) in an external stationary electromagnetic field we have to do the following:

- to write the classical Lorentz equation for the $K$-particle and
- to replace the 4-velocity of the $K$-particle, $(0) u^\mu$, in the resulting equation in accordance with Eq.(12).

Equation (12) possesses another important feature. If the classical velocity of the $K$-particle, $(0) u^\mu$, can be found as a function of the coordinates $(0) u^\mu = (0) u^\mu(x^i)$ (i.e., the $K$-particle velocity field is determined) without using the $K$-motions equations, this relation written as a differential equation,

\begin{equation}
dx^i / d\tau = (0) u^i(\tau) + g^i i k_i/c ,
\end{equation}

(14)
is equivalent to the $K$-motions equations (11).

It should be emphasized that this result has already been obtained for potential fields in [1].

4. The $K$-field equation written as (1) was derived assuming that the mixed components of the metric tensor of the manifold $k V_4$, are not equal to zero: $(k) g_{0i} \neq 0$.

The variety $k V_4$ itself, however, was constructed in [1] as a set of isotopic surfaces, $k G_{04} \subset k V_4$, of the 4-dimensional space with the metrics

\begin{equation}
(k) ds^2 = (k) g_{0i}^2 dt^2 + g_{ij}^i d^i d^j .
\end{equation}

(15)

It is this feature of the manifold $k V_4$ which allowed us to consider the formalism of the $K$-field theory as the generalization of de Broglie postulate. This means that on the manifold $k V_4$, ordinary test particles behave in the same fashion as photons in the Minkovskii space $V_4$, i.e., they move along the isotropic geodesic lines.

That is to say that, assuming the components $(k) g_{0i} \neq 0$, we do, in fact, reject de Broglie’s postulate as the fundamental theoretical postulate. Therefore, for the logic of the $K$-field formalism to conform to the fundamental postulate, we have to take the components of the mixed metric tensor of the manifold $k V_4$ to be equal to zero, $(k) g_{0i} = 0$.

Then, the solutions to the $K$-field equation in the $K$-system, as obtained in [1], will have the following form (in a nonrelativistic approximation):

\begin{equation}
k_0(x^i, t) = R_0(x^i) \sin(\omega t + \varphi_0) ,
\end{equation}

(16)

\begin{equation}
k_3(x^i, t) = (c/\omega) D_3 R_0(x^i) \cos(\omega t + \varphi_0) ,
\end{equation}

(17)