STRESS-STRAIN STATE OF A THREE-LAYER METAL-CONTAINING POLYMERIC ROD

É. I. Starovoitov and A. V. Yarovaya  

We consider a three-layer metal-containing polymeric rod subjected to thermal and mechanical loads taking into account the physical nonlinearity and rheonomic properties of the materials of the layers. For this rod, we obtain an exact solution of the problem of the theory of elasticity. For the solution of the problem of thermoviscoelastoplasticity, we apply the method of "elastic" solutions and present the results of its numerical realization.

In the present work, we consider a three-layer rod that is asymmetric across its thickness. Its external load-carrying layers are made of a metal and the internal layer (filler) is a polymer that is incompressible in the transverse direction. For the description of the kinematics of this sandwich, we use the hypotheses of the broken normal, i.e., the Kirchhoff hypotheses are assumed to be true in the load-carrying layers, whereas in the filler, the normal remains rectilinear, does not change its length, and rotates by an angle \( \psi(x, t) \). We assume that the load-carrying layers are viscoelastoplastic and the filler is nonlinearly viscoelastic.

Statement of the Boundary-Value Problem. We relate a coordinate system \( x, y, z \) to the midplane of the filler (Fig. 1). Distributed forces \( p(x, t) \) and \( q(x, t) \) act upon the external layer of the rod. By \( w(x, t) \) and \( u(x, t) \) we denote the deflection and longitudinal displacement of the midplane of the filler, respectively.

The longitudinal displacements in the layers \( u^{(k)} \), where \( k = 1, 2, 3 \) is the number of the layer, can be expressed via three unknown functions \( u(x, t), \psi(x, t), \) and \( w(x, t) \) as follows:

\[
\begin{align*}
    u^{(1)} &= u + c\psi - zw_{,x}, \quad c \leq z \leq c + h_1, \\
    u^{(3)} &= u + z\psi - zw_{,x}, \quad -c \leq z \leq c, \\
    u^{(2)} &= u - c\psi - zw_{,x}, \quad -c - h_2 \leq z \leq -c,
\end{align*}
\]

where \( z \) is the distance from a fiber under consideration to the midplane of the filler and commas in the subscripts denote the operation of differentiation with respect to the variable standing immediately after the comma. The components of the strain tensor in the layers can be determined by using the well-known Cauchy relations.

Internal forces and moments are introduced as follows:

\[
N^{(k)} = b \int_{h_k}^{h} \sigma_x^{(k)} \, dz, \quad M^{(k)} = b \int_{h_k}^{h} \sigma_x^{(k)} z \, dz, \quad \text{and} \quad Q^{(3)} = b \int_{h_3}^{h} \sigma_{xx}^{(3)} \, dz,
\]

where \( \sigma_x^{(k)}, \sigma_{xx}^{(3)} \) are components of the stress tensor and \( b \) is the width of the cross-section. In (2), the integrals are taken over the height of the \( k \)th layer.

The equilibrium equations can be obtained by using the virtual work principle, i.e.,

\[
\delta A_e + \delta A_t = 0,
\]

where $\delta A_e$ is the work of external forces and $\delta A_i$ is the work of internal elastic forces. In determining the work of external forces, we assume that the external surfaces of the load-carrying layers are subjected to the action of arbitrarily distributed loads. At the same time, the forces and moments $N_p$, $M_p$, and $Q$ are applied to the end faces of the rod.

We represent the work of external surface loads in the form

$$\delta A_e = \iint_S (p\delta u + q\delta w)dS = b\int_0^l (p\delta u + q\delta w)dx.$$  \hspace{1cm} (4)

At the same, in view of (1) and (2), the work of elastic forces is given by the formula

$$\delta A_i = \iiint_S \sum_{k=1}^3 \iint_{h_k} (\sigma_{x}^{(k)}\varepsilon_{x}^{(k)} + \sigma_{xz}^{(k)}\varepsilon_{xz}^{(k)})dxdS = \iiint_S \left(N\delta u_x - M\delta w_{,xx} + Q\delta \psi + H\delta \psi_x\right)dS,$$  \hspace{1cm} (5)

where $N = \sum_{k=1}^3 N^{(k)}$, $M = \sum_{k=1}^3 M^{(k)}$, $Q = Q^{(3)}$, and $H = c\left(N^{(1)} - N^{(2)}\right) + M^{(3)}$.

If we now substitute (4) and (5) in (3), then, after simple transformations, we arrive at the equilibrium equations

$$bp - N_{,x} = 0, \quad Q - H_{,x} = 0, \quad \text{and} \quad bq - M_{,xx} = 0$$  \hspace{1cm} (6)

with the following boundary conditions for forces:

$$N = N_p, \quad M = M_p, \quad M_{,x} = Q, \quad \text{and} \quad N = 0 \quad \text{for} \quad x = 0, l,$$

where

$$N_p = \int_{-c-h_2}^{c+h_2} \sigma_p dz, \quad M_p = \int_{-c-h_2}^{c+h_2} \sigma_p zdz,$$

$\sigma_p = \sigma_0$ for $x = 0$, $\sigma_p = \sigma_l$ for $x = l$, and $\sigma_0$ and $\sigma_l$ are given stresses on the ends of the rod. We assume that the ends are covered with rigid membranes used to suppress relative displacements of the layers.

In the layers, we use the following hereditary nonlinear physical equations of state:

$$S_{ij}^{(k)} = 2G^{(k)}(T^{(k)}) \left[ f_1^{(k)}(\varepsilon_u^{(k)}, T^{(k)}) \ddot{e}_{ij}^{(k)} - \int_0^t \Gamma_k(t-\tau) f_2^{(k)}(\varepsilon_u^{(k)}, T^{(k)}) \dot{e}_{ij}^{(k)}(\tau) d\tau \right],$$

$$\sigma^{(k)} = 3K^{(k)}(T^{(k)}) (\varepsilon^{(k)} - \alpha_{\theta k} \Delta T^{(k)}), \quad k = 1, 2, 3,$$  \hspace{1cm} (7)