DISTRIBUTION OF TEMPERATURE IN A PLATE
WITH A SINGLE-LAYER COATING SUBJECTED TO INTENSE HEATING*

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We establish the distribution of temperature formed under conditions of abrupt changes in the surface temperature near the adhesional contact between the base and the coating in a plate of infinite length made of molybdenum or niobium both sides of which are covered with silicide coatings. The problem of heat conduction for a multilayer plate subjected to thermal cycling is solved by the method of finite integral transformations. It is shown that a silicide coating with a thickness of 60–100 μm leads to the formation of a significant temperature gradient in the base material under conditions of cyclic variation of temperature.

Recent years are marked by the appearance of numerous works devoted to the analysis of the behavior of materials with coatings under conditions of thermal cycling (see [1–3]). Since it is methodologically difficult to create reliable experimental procedures capable of determination of temperature gradients formed in materials with coatings under conditions of abrupt changes in temperature, these gradients can be evaluated in the first approximation by using numerical methods.

In the present work, we determine the nonstationary distribution of temperature formed under conditions of nonstationary changes in surface temperature near the interface of the base and the coating in a plate of infinite length made of molybdenum or niobium both sides of which are covered with silicide coatings.

Under conditions of nonstationary changes in temperature, temperature gradients can be evaluated by solution of the problem of heat conduction for a multilayer plate (Fig. 1). This problem has been studied by many authors (see [4–7]).

To solve this problem in analytic form, one may apply either the Laplace transformation with respect to time or the method of separation of variables. In the first case, we arrive at a nontrivial problem of construction of the inverse Laplace transformation. At the same time, the procedure of direct separation of variables requires homogeneous boundary conditions.

The method of finite integral transforms suggested in [8,9] is now extensively used for the solution of applied problems of the indicated type. It is reasonable to use this method for the solution of problems of heat conduction for multilayer plates under conditions of nonstationary changes in temperature because it enables one to apply the same algorithm for all types of boundary conditions.

As boundary conditions, we take the conditions of heat exchange with mean variable temperature specified on the corresponding surfaces of the specimen (Fig. 1), namely,

\[
\lambda_i \frac{\partial T(x_i, r)}{\partial x} - \alpha_i \left[ T(x_i, r) - T_c_i(r) \right] = 0,
\]

\[
\lambda_m \frac{\partial T(x_m, r)}{\partial x} + \alpha_2 \left[ T(x_m, r) - T_c_2(r) \right] = 0,
\]

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where \( x_i - x_0 = h_i, \; x_2 - x_1 = h_2, \ldots, \; x_m - x_{m-1} = h_m, \; h_j \) is the thickness of the \( j \)th layer, \( T_{c_1} \) and \( T_{c_2} \) are ambient temperatures, \( \alpha_1 \) and \( \alpha_2 \) are the heat-transfer coefficients, and \( \lambda_j \) is the thermal conductivity of the \( j \)th layer.

As initial conditions, we take

\[
T(x_0, 0) = T(x_m, 0) = T(x_j, 0) = T_{st},
\]

where \( T_{st} \) is the stationary temperature in the interior of the specimen. To determine the distribution of temperatures over the thickness of the plate, it suffices to restrict ourselves to the condition of perfect thermal contact and ignore the possibility of discontinuities. In this case, the problem of determination of the kernel of the integral transformation is reduced to the problem of eigenvalues of a homogeneous equation corresponding to the original inhomogeneous equation with homogeneous boundary conditions [9].

Thus, in particular, to find the kernel of the integral transformation for a multilayer system, it is necessary to solve the Sturm–Liouville problem for an equation with piecewise-constant coefficients and boundary conditions of the third kind. This problem can be solved by the method of conjugation [8]. In this case, the kernel of the integral transformation can be found relatively easily and the problem of determination of the distribution of temperatures over the cross section of the specimen reduces to the operations of formal summation and elementary integration.

In the presence of heat sources, the required distribution of temperatures satisfies the differential equation

\[
\frac{\partial T}{\partial \tau} = \frac{1}{C(x)\rho(x)} \frac{\partial}{\partial x} \left[ \lambda(x) \frac{\partial T}{\partial x} \right] + W(x, \tau)
\]

(2)
everywhere in the interval except the discontinuity points of the coefficients of this equation. Here, \( C(x) \) is specific heat, \( \rho(x) \) is density, and \( W(x, \tau) \) is the intensity of heat sources.

Finally, the solution of the considered problem obtained by the method of finite integral transformations can be represented in the form

\[
T(x, \tau) = \sum_{n=1}^{\infty} \frac{K_n(x)}{||K||} e^{-\mu_n^2 \tau} \left[ \theta_n + \int_0^\tau Q_n(t) e^{\mu_n^2 t} dt \right],
\]

(3)

where

\[
Q_n(t) = \alpha_2 K_n(x_m)T_{c_1}(t) + \alpha_1 K_n(x_0)T_{c_2}(t) + \int_{x_0}^{x_m} W(x, \tau) C(x) K_n(x) \rho(x) dx,
\]