ATMOSPHERIC REFRACTION
ABOVE THE INLAND ICE IN NORTH GREENLAND*

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INTRODUCTION
The method and results of trigonometrical levelling, carried out across the inland ice of North Greenland as part of the work of the British North Greenland Expedition, have been described elsewhere. Determination of the height difference between successive survey stations requires simultaneous measurement of the vertical angle from each station to the other. From these simultaneous reciprocal vertical angles the refraction of the light ray by the layers of the atmosphere through which it passes can be calculated. Meteorological observations were made in addition. The purpose of the present paper is to describe the variations in refraction and to correlate them with the meteorological conditions.

METHOD
Vertical angles were observed from one station directly on to the theodolite at the other station. It did not prove possible to sight on to a theodolite with sufficient accuracy when an observer was standing behind it, and thus, strictly speaking, the reciprocal vertical angles were not measured simultaneously. The time interval was reduced to a minimum however, being never more than fifteen minutes and generally about ten (ten minutes between the times when the observers at the two stations started to measure angles). The measured refraction is thus an average value for a period of ten minutes. It must be assumed that refraction is the same at each end of the path, although this may not be strictly true on some occasions.

The refraction coefficient, \( k \), is calculated from the formula

\[
2k = 1 - (z + z' - 180^\circ) \sin \left( \frac{1^\circ}{r/x} \right)
\]

where \( z, z' \) are the measured zenith distances, \( r \) is the earth's radius inclusive of height of station above mean sea level, and \( x \) is the distance between stations. The average value of \( x \) is about 4 kilometres. The standard errors of \( z, z' \) are one second.

Theodolites were always between 1.3 and 1.75 metres above the snow surface. The greatest and least values of observed refraction coefficient were +0.9 and -0.25. Taking a distance of 5 kilometres between stations, which is greater than average, and assuming that the surface between stations is of uniform slope, this implies that the light ray was always between 1.0 and 2.6 metres above the snow surface. All conclusions regarding refraction and temperature gradient refer to this layer of the atmosphere.

The work was carried out during May, June, and July of 1953 and 1954.

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In North Greenland, the sun is always above the horizon in these months. Meteorological observations were made twice daily during the first year. In the second, they were made immediately after each set of vertical angles had been measured. Air temperature was measured at each end of the line, and the average value has been used; but other observations were made only at one end. Air temperatures were measured, in the shade, about 0.75 metre above the surface, thermometers being read to the nearest degree (Fahrenheit). An aircraft aneroid was used to measure atmospheric pressure and the standard error was 1 millibar. Wind speed was measured by hand anemometer about 4 metres above the surface. Amount and type of cloud, and presence of drifting snow were noted in addition. Humidity was not measured, but observations made at the Expedition's station on the inland ice showed that the mean humidity was roughly 80 per cent and did not vary greatly. At the low temperatures encountered, this is equivalent to a vapour pressure of not more than 3 millibars so, for most purposes, the air can be regarded as dry.

Near the edges of the inland ice, the surface has marked undulations, and usually survey stations were located on the crests. In this case the height of the light ray above the surface changes considerably, and the values of refraction show marked variations which are difficult to interpret. For this reason, detailed analysis has been confined to results from the central area of the inland ice, where the assumption of uniform surface slope between stations is considered to be justified. The analysis has been restricted further to work done during the second year, when meteorological observations were made each time vertical angles were measured.

In this restricted region, the inland ice slopes down very gradually towards the west. The maximum slope between adjacent stations was 15 minutes; the greatest height difference was 19 metres. The altitude lay between 2000 and 2550 metres. The distance between stations ranged from 2.8 to 5.4 kilometres, with a mean of 4.8 kilometres. The work was done in late May and June 1954. It is considered that surface conditions and other features, which might affect refraction but which cannot be measured, are comparatively uniform in this region.

Eighty-five values of refraction coefficient and meteorological observations are available, at least three and sometimes four or five on each day.

The range of temperature was -5°F to +26°F, of pressure, 730 to 790 millibars, and the wind speed ranged from 0 to 20 knots with a mean of 11 knots.

THEORY

The theory in this section broadly follows that given by Bomford².

The equation for change of density ρ with height h in the atmosphere is

\[
\frac{\partial \rho}{\partial h} = -\frac{\rho}{RT^2} \left( g \frac{\partial T}{\partial h} + \frac{\partial T}{\partial h} \right)
\]

where ρ is the pressure, T the absolute temperature, g the acceleration due to gravity, and R the universal gas constant.

The refractive index μ of air is related to the density by the equation

\[\mu - 1 = c\rho\]

where c is a constant.