AZIMUTH DETERMINATION BY EQUATORIAL STARS*

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To determine precise azimuths it is customary to make observations on Polaris, the information recorded for each pointing being the time of pointing, the stride level reading, and the horizontal circle reading. Polaris has been regarded as the most suitable star to observe because its slow motion around the Pole makes it easy to set on and makes the recorded time of observation not too critical. As work proceeds farther northward, however, the altitude of Polaris increases, and the effects of stride level and pointing errors increase respectively as the tangent and secant of the altitude. The increase of these errors with altitude suggests that, for higher latitudes, azimuth observations might better be made on stars closer to the equator. The time measurements, of course, become more critical for stars nearer the equator, but the use of an impersonal micrometer in conjunction with a recording chronograph should satisfy this requirement.

Accordingly, a method was designed to observe stars with an astronomic transit telescope as they cross the plane defined by the reference mark and the vertical through the observation station. From these observations, the azimuth of the reference mark may be computed. The method was tried experimentally by the Geodetic Survey of Canada during the summer of 1954 with encouraging results.

I

TIME OF STAR CROSSING

We may consider the error in the observed time of a star crossing in two parts: (a) the recording error, depending on the method of timing, whether it be by chronograph, stop-watch, or eye and ear; and (b) the pointing error, being the effect of the failure to keep the cross-hair exactly on the star. The probable error, $m$, of one determination of the time of star crossing may be given as

$$m^2 = a^2 + b^2/(V^2 \cdot P^2) \ldots \ldots (1)$$

where the magnitudes of $a$ and $b$ depend on the particular timing method, observer, and transit instrument; $V$ is the velocity component of the star perpendicular to the cross-hair; and $P$ is the magnification of the telescope.

If the time of crossing is determined on $n$ wires or on a movable cross-hair

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with \( n \) electrical contacts, the probable error of the mean observed time will be given as

\[
M_T^2 = \frac{1}{n} \left( \frac{a^2}{V^2} + \frac{b^2}{P^2} \right) \quad \ldots \quad (2)
\]

T. Niethammer gives the following numerical values for \( a \) and \( b \) based on probable errors received in various methods of observing:

<table>
<thead>
<tr>
<th>Method</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eye and ear</td>
<td>( \pm 0.067 )</td>
<td>( \pm 3.17 )</td>
</tr>
<tr>
<td>Manual recording</td>
<td>0.047</td>
<td>3.17</td>
</tr>
<tr>
<td>Impersonal micrometer—</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Geodetic Institute in Potsdam</td>
<td>0.038</td>
<td>2.02</td>
</tr>
<tr>
<td>(ii) Swiss Geodetic Commission</td>
<td>0.021</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Putting these values of \( a \) and \( b \) in the expression (2) shows that for a good approximation we may rewrite

\[
M_T = \frac{M}{(V \cdot P)} \quad \ldots \quad (3)
\]

where \( M \) is a constant for given values of \( a \) and \( b \).

The velocity component, \( V \), is given by the expression

\[
V = \frac{da}{dt} \sin z = \cos \rho \cdot \cos \delta \quad \ldots \quad (4)
\]

where \( \frac{da}{dt} \) is the rate of change of azimuth with time, \( z \) is the zenith distance, \( \rho \) is the parallactic angle and \( \delta \) is the declination of the star.

Substituting the value of \( V \) from (4) into expression (3) gives

\[
M_T = \frac{M}{(P \cdot \cos \rho \cdot \cos \delta)} \quad \ldots \quad (5)
\]

From (5) we see that the probable error, \( M_T \), reaches a maximum when \( \cos \rho \cdot \cos \delta = 0 \), or when the star is observed at elongation.

MEASUREMENT OF THE HORIZONTAL ANGLE

It was stated in the introduction that the time of star crossings would be observed in the azimuth plane of the reference object. It is, of course, not possible to set the transit instrument perfectly in this plane; in practice, the transit is set as closely as is practicable to the vertical plane through the reference object, and the angle between this plane and the collimation plane of the telescope is measured by repeated micrometer readings taken on the reference object with the telescope in the forward and reverse position.

The error, \( M_A \), in measuring this angle will spring from the errors \( M_p \), the pointing error, and \( M_R \), the reading error. Since the reference object will usually be in the horizontal plane, the reading and pointing errors will enter into the horizontal angle error directly, so that

\[
M_A^2 = M_p^2 + M_R^2
\]