ON TESTING THE EQUALITY OF PARAMETERS IN k TRIANGULAR POPULATIONS WITH UNEQUAL OBSERVATIONS*

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1. Introduction

Murty [4] gave the distribution of the quotient of maximum values in samples from a rectangular distribution and used it to test the hypothesis that the two samples come from the same rectangular distribution. Rider [5] gave the statistic the ratio of two ranges for testing the equality of rectangular populations. Khatri [2], [3] has extended these results for testing the equality of ranges of k-rectangular populations. Bartos [1] has given a test to test the equality of scale parameters in k-symmetrical triangular populations given by

\[ f_i(x) = \begin{cases} a_i^{-i}(1-a_i^{-1}|x-b_i|) & \text{for } |x-b_i| \leq a_i \\ 0 & \text{otherwise} \end{cases} \]

\[ i=1, 2, \ldots, k, \] where \( b_i (i=1, 2, \ldots, k) \) are known or zero. The test developed by him is as follows:

Reject \( H_0(\alpha_1=\alpha_2=\cdots=\alpha_k) \) if \( v=(d_{\min}/d_{\max}) \leq c \) where \( d_{\min}=d_{\min}(x_{ij}-b_i) \), \( d_{\min}=\min d_{\min} \), \( d_{\max}=\max d_{\max} \), \( x_{ij} (j=1, 2, \ldots, n_i) \) are independent observations from \( f_i(x) \), and \( c \) is given by

\[ P\{v \leq c/H_0(\alpha_1=\alpha_2=\cdots=\alpha_k)\} = \alpha . \]

Bartos [1] has given the null distribution of \( v \) when \( n_1=n_2=\cdots=n_k=n \) (say) and the power function of the test is derived for \( k=2 \) and \( n_1=n_2 \) only. In this paper, we obtain the null distribution of \( v \) when \( n_i \)'s are unequal and the power function of the test procedure up to \( k=3 \) only. The general case is complicated, and so it is not given. The lower

* The authors derive the null and non null distributions of the statistic \( v=d_{\min}/d_{\max} \) \((d_{i}=\max |x_{ij}-b_i|, \ d_{\min}=\min d_i, \ d_{\max}=\max d_i, \ j=1, 2, \cdots, n \quad \text{and} \quad i=1, 2, \cdots, k) \) connected in testing the equality of scale parameters in k-symmetrical triangular populations when \( b_i, b_1, b_2, \cdots, b_k \) are known. The null case is considered for any \( k \) and non-null up to \( k=3 \). The similar work when \( b_i \)'s are unknown will be given in due course of time.
5% points of this test are tabulated for \( k=2 \) populations and are given at the end of the paper.

The problem for testing \( H_0(a_1=a_2=\ldots=a_k) \) when \( b_i \)'s are unknown has been solved and will be published in due course of time.

2. Null distribution of the quotient of maximum values

Let \( d_{(i)} \) denote the \( \max |x_i-b_i| \) in a random sample of size \( n \) from the population (1) when \( a_i=a \) (say), then the density function of \( d_{(i)} \) is given by

\[
2n_i \left( \frac{1 - d_{(i)}}{a} \right) \left( \frac{2d_{(i)} - d_i^2}{a} \right)^{n_i-1} \quad \text{for } 0 < d_{(i)} < a, \ i=1, 2, \ldots, k
\]

and by 0 otherwise. Let \( d_{(i)} \)'s be ordered as \( d_1 \leq d_2 \leq \cdots \leq d_k \). Then the joint density function of \( d_1,d_2,\ldots,d_k \) is given by

\[
g(d_1,d_2,\ldots,d_k) = \frac{2^k n_1 n_2 \cdots n_k}{a^N} \sum_{\phi} \left( \frac{1 - d_1}{a} \right)^{n_1} \left( \frac{2d_2 - d_1^2}{a} \right)^{n_2-1} \cdots \left( 1 - \frac{d_k}{a} \right)^{n_k-1}
\]

where \( \sum \) denotes the summation over \( (\phi_1,\phi_2,\ldots,\phi_k) \), the permutations of \( (1,2,\ldots,k) \) and \( N=n_1+n_2+\cdots+n_k \). From this, it is easy to see that

\[
P(v \leq c) = P(d_1 \leq cd_k) = 0, \quad \text{for } c \leq 0,
\]

\[
H_k(c) = \begin{cases} 
\int_0^a \int_{d_1}^{cd_k} \int_{d_2}^{d_1} \cdots \int_{d_k}^{d_{k-1}} g(d_1,\ldots,d_k) \, dd_1 \cdots dd_k, & \text{for } 0 \leq c \leq 1, \\
1 & \text{for } c \geq 1,
\end{cases}
\]

or

\[
H_k(c) = \begin{cases} 
0 & \text{for } c \leq 0, \\
\int_0^1 \int_0^{cx} \int_0^{x_1} \int_0^{x_2} \cdots \int_0^{x_{k-2}} g(ax_1,ax_2,\ldots,ax_{k-1}) \, dx_1 \cdots dx_{k-1}, & \text{where } x=d_k/a \text{ or } d_k=ax, \text{ for } 0 \leq c \leq 1, \\
1 & \text{for } c \geq 1.
\end{cases}
\]

First of all, let us consider the particular cases:

(i) Let \( k=2 \). Then integrating w.r.t. \( x_1 \), we get