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LONG LINES—TRILATERATION ADJUSTMENT

As a result of discussion by a small group particularly interested in these matters while at the Rome 1954 Assembly, it was decided to continue the work of the Group with essentially the same composition of membership. Time has altered this intention, nevertheless former members and those since added are vitally concerned in these matters, as are many organizations in addition to those purely geodetic. The literature has thus been greatly increased in the last three years and many new publications are available for further study by this group.

The reports from various contributors have been summarized as follows:

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Mr. Rainsford

Mr. Braziër has discussed with me the project for examining all known formulae for computing lines (both short and long) on the spheroid, and setting out the errors involved. My examinations of both the Clarke and Mid-Latitude formulae have been published in the E.S.R. and I do not think we could find the time required for similar investigations of the many other formulae involved.

My investigation of Long Lines was published earlier and in the end of that article I made three suggestions for work which should be done for this project.

(1) Further examination to determine where Sodano's formula breaks down and what are the errors.

(2) Computation and publication of Army Map Service tables of A, B, C, D, E, F, factors for the Clarke 1866 Figure of the Earth.

(3) Computation and publication of Levallois-Dupuy tables (Wallis Integrals) in the sexagesimal system.

I think the Americans said that they were going to do (1) and (2) but so far I have not seen any results. I feel sure that these three projects would be of great value to the geodetic world.

With regard to Long Geodesics on the Ellipsoid, it should be noted that there is one peculiarity of geodesics which approach 180° of arc or halfway round the world. Take the case of two points on the equator, 180° apart in
longitude. If these two points were on a sphere, it is obvious that there would be an infinite number of great circles through both points, and all of the same length. On the spheroid, however, there are only two geodesic arcs, corresponding to great circles, which pass through two points on the equator, exactly 180° apart. But the meridian arc is shorter than the equatorial arc through the two points, and it is, therefore, the only true geodesic on the shortest line definition of a geodesic. Consequently, if two points are situated near the equator and are separated by nearly 180° of longitude there is a certain ambiguity as to what is meant by the geodesic between them. There is a full discussion of this point in an article 'The distance between two widely separated points on the surface of the earth.' This article is a review of another article of the same title by Mr. W. D. Lambert.4

In the past, the problem of long lines on the earth has frequently been approached by the use of plane sections, since the geometry was (supposedly) easier to understand than that of geodesic curves in three dimensions. The difference in length between a geodesic and a plane curve is very small, but the difference in azimuth can be quite appreciable for lines over about 500 miles. If, for example, it were known that radar waves followed a plane section round the world, there might be some justification for plane section computation; in the absence of evidence that this is the case, the geodesic approach is theoretically more correct and actually simpler in practice. It seems probable that the method advocated by Levallois and Dupuy for computation of long lines is the best, provided that tables are made for various Figures of the Earth in sexagesimal units.

For lines less than 500 miles (800 km), in latitudes less than 75°, formulae are available using either plane curves or geodesics.5 The first two methods are based on plane curves; they will give precise results but require the use of 9 or 10-figure trigonometric tables. The other methods advocated are based on true geodesics and precise results can be obtained with 8-figure tables. For the inverse problem, the Mid-Latitude Formulae are undoubtedly the best as they do not require any successive approximation: the extension to fourth order corrective terms has been given in 6. For the direct problem the extension of Clarke's approximate formulae may be used: corrective terms have been given up to 6th order (spherical) and 5th order (elliptic), but many of these are inappreciable except in high latitudes. Formulae have also been given for the Puissant Series method up to 7th order (spherical) and 6th order (elliptic) terms. It is not recommended that these should be used for long lines as they do not converge sufficiently rapidly, but they are useful to obtain the possible errors involved in neglecting terms of any particular order.

The best geodetic tables available are those for Latitude Functions on the various spheroids (natural values of the meridional arc; A, B, C, D, E and F factors; radii of curvature, R and N) produced by the Army Map Service in 1944. They tabulate the required functions to an accuracy of a millimetre at an interval of 1° of arc. These tables are invaluable for any form of geodetic line computations: they are clear, well set out and very easy to use as differences are given for a 1″ interval. It seems strange that the A.M.S. computed these tables for every spheroid in common use, except their own (the Clarke 1866). If they could now see their way to producing the 1866