NECESSARY AND SUFFICIENT CONDITIONS FOR THE ABSOLUTE STABILITY OF DISCRETE TYPE LURIE CONTROL SYSTEM

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Abstract

In this paper, it is discussed that the absolute stability for zero solution of the discrete type Lurie control system

\[
\begin{align*}
x(n+1) &= Ax(n) + bf[\sigma(n)] \\
\sigma(n) &= c^T x(n)
\end{align*}
\]

in which the nonlinear function \( f(\sigma) \) satisfying conditions as follows

\[
\begin{align*}
f(0) &= 0, \quad \sigma(\sigma) > 0 \quad (\sigma \neq 0) \quad (2) \\
\text{or} \\
f(0) &= 0, \quad 0 \leq k_1 \leq f(\sigma)/\sigma \leq k_2 < +\infty \quad (\sigma \neq 0) \quad (3)
\end{align*}
\]

It gives the necessary and sufficient conditions for the absolute stability for system (1) under conditions (2). We also obtain the sufficient criteria for absolute stability of the simplified system of (1) under conditions (3).

Key Words: discrete type, absolute stability, necessary and sufficient condition, Lurie problem

I. Introduction

More than forty years ago, the Pro-Soviet Union author, A. I. Lurie proposed the Lurie control system and Lurie problem[1], which have general significance in the nonlinear control theory and control engineering, by investigating the stability of automatic operating instrument of aircraft. Since that time, many authors have extensively studied Lurie control systems to describe in various forms and obtained a lot of results for absolute stability[2-4]. Unfortunately, it was only obtained the sufficient criteria for absolute stability of Lurie control system or necessary and sufficient conditions for some special classes[5-8]. Up to now, there is not a complete and constructive result for Lurie control systems to describe in various forms.

In Refs. [9-11], the absolute stability of zero solution is investigated for the discrete type Lurie control systems as follows

\[
\begin{align*}
x(n+1) &= Ax(n) + bf[\sigma(n)] \\
\sigma(n) &= c^T x(n)
\end{align*}
\]
where matrix $A = (a_{ij})_{m \times m} \in \mathbb{R}^{m \times m}$, vectors $x, b, c \in \mathbb{R}^m$, nonlinear function $f(\sigma)$ satisfying conditions as follows

$$f(0) = 0, \; \sigma f(\sigma) > 0 \quad (\sigma \neq 0) \quad (1.2)$$
or

$$f(0) = 0, \; 0 \leq k_1 \leq f(\sigma)/\sigma \leq k_2 < +\infty, \quad (\sigma \neq 0) \quad (1.3)$$

in [9], Liao Xiaoxin obtained the necessary and sufficient conditions and some sufficient algebraical criteria for the absolute stability of zero solution of system (1.1) under conditions (1.2) (called infinite sector condition) or conditions (1.3) (called finite sector condition). The criteria are available under finite sector condition, but they are unavailable under infinite condition except some special cases. So are results in [10, 11].

In this paper, firstly, we establish the dimension reducing principle. By using the principle, we obtain the explicitly necessary and sufficient condition for the absolute stability of system (1.1) under infinite sector condition. We also give some sufficient algebraical criteria for system (1.1) under finite sector condition.

II. Necessary and Sufficient Conditions for Absolute Stability of System (1.1) under Infinite Sector Condition

We establish the dimension reducing principle first. Similar to [12], we obtain the Lemma as follow

**Lemma 1** If $\text{Rank} \{c, A^Tc, \cdots, (A^{m-1})^Tc\} = m_1$, then there exists a nonsingular linear transformation $x = My$, such that system (1.1) can be transformed into

$$y(n+1) = \bar{A}y(n) + \bar{b}f(\sigma(n))$$
$$\sigma(n) = \bar{c}^T y(n)$$

where

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \cdots \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \end{bmatrix}, \quad \bar{c} = \begin{bmatrix} \bar{c}_1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$\bar{A}_{11} \in \mathbb{R}^{m_1 \times m_1}, \; \bar{A}_{12} \in \mathbb{R}^{m_1 \times m_2}, \; \bar{b}_1, \; \bar{c}_1, \; y_1 \in \mathbb{R}^{m_1}, \; \bar{b}_2, \; y_2 \in \mathbb{R}^{m_2}, \; m_1 + m_2 = m$, and

$\text{Rank} \{\bar{c}_1, \bar{A}_{11}^T \bar{c}_1, \cdots, (\bar{A}_{11}^{m_1-1})^T \bar{c}_1\} = m_1$

**Theorem 1** If $\lambda$ is Schur stable, namely spectral radius $R(\lambda) < 1$, then the absolute stability of zero solution of system (1.1) is equivalent to that of subsystem of (2.1)

$$y_1(n+1) = \bar{A}_{11} y_1(n) + \bar{b}_1 f_1(\sigma(n))$$
$$\sigma_1(n) = \bar{c}_1^T y_1(n)$$

**Proof** Because the transformation $x = My$ is nonsingular, the absolute stability of system (1.1) is equivalent to that of system (2.1). It is obvious that the absolute stability of zero solution of system (2.1) implies that of system (2.2). Similar to the proofs of [12], it is easy to prove that if zero solution of system (2.2) is absolute stable, then zero solution of system (2.1) is absolute stable, that is, zero solution of system (1.1) is absolute stable. The proof is complete.

Lemma 1 and Theorem 1 construct the reducing dimension principle. It is obvious that the reducing dimension principle is true under finite sector condition. Contrasting with Ref. [11], the principle in this paper is explicit and more convenient for application.