PLASTIC BUCKLING OF STIFFENED TORISPHERICAL SHELL

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Abstract

This paper uses the nonlinear prebuckling consistent theory to analyse the plastic buckling problem of stiffened torispherical shell under uniform external pressure. The buckling equation and energy expressions of the shell are built, the calculation formula is presented. Numerical examples show that the method in this paper has better precision and the calculating process is very simple.

Key words stiffened shell of revolution, torispherical shell, uniform external pressure, plastic buckling

I. Introduction

Stiffened torispherical shell is a shell of revolution, which consists of spherical shell and a hoop shell, it is often used in engineering as a head of vessels and submarine, and component of missiles. The shell must bear uniform external pressure and buckling is one of the main forms of collapse. In engineering the critical pressure is often estimated by the formula of stiffened spherical shell and it shall produce some errors obviously. It shall take a lot of calculation expenses and times and it is not suitable in the primary design. This paper uses the nonlinear prebuckling consistent theory to analyse the plastic buckling of this kind of shell, the calculation formula is presented on the energy principle, the method and formula can solve the buckling problems of stiffened spherical shell and stiffened torispherical shell.

II. Basic Equations

The basic form of torispherical shell discussed in the paper is shown in Fig. 1.

Fig. 1. The basic form of the shell

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The strain relations of shell can be written from Ref. [1]:

\[
e_{\varphi} = e_{\varphi}^{(1)} + e_{\varphi}^{(2)} = \left\{ u' + \frac{u}{R_1} \right\} + \left\{ \frac{1}{2} (\beta^2 + \psi^2) \right\}_t + \left\{ 0.5(\beta^2 + \psi^2) \right\}
\]

\[
e_{\theta} = e_{\theta}^{(1)} + e_{\theta}^{(2)} = \left\{ \frac{u}{r} + u' \right\} + \left\{ \frac{1}{2} (\psi^2 + \varphi^2) \right\}
\]

\[
e_{\varphi\theta} = e_{\varphi\theta}^{(1)} + e_{\varphi\theta}^{(2)} = \left\{ \frac{u}{r} + r \left( \frac{u'}{r} \right) \right\} + \{ \beta \psi \}
\]

\[
k_{\varphi} = \beta', \quad k_{\theta} = \psi - \frac{r'}{R_2}
\]

where,

\[
r = R_1 \sin \varphi, \quad \beta = \frac{u}{R_1} - u', \quad \psi = \frac{v}{R_2} - \psi', \quad \varphi = \frac{1}{2} \left( -\frac{\dot{u}}{r} + u' + \frac{ur'}{r} \right)
\]

Superscript (1) and (2) are respectively expressed as linear and nonlinear part in the strain. \( R \) and \( R_2 \) are respectively meridional and circumferential of curvature.

Assuming prebuckling state is "c" and buckling state is "p", the strain relations before buckling can be obtained by taking prebuckling displacement pattern into formula (2.1) and the strain relation can be shown as when buckling:

\[
e_{\varphi}^{(p)} = e_{\varphi}^{(1)} + e_{\varphi}^{(2)} + e_{\varphi}^{(p)} = \left\{ u' + \frac{u}{R_1} \right\} + \left\{ \frac{1}{2} (\beta^2 + \psi^2) \right\}_t + \{ \beta \psi + \beta \psi' \}
\]

\[
e_{\theta}^{(p)} = e_{\theta}^{(1)} + e_{\theta}^{(2)} + e_{\theta}^{(p)} = \left\{ \frac{u}{r} + u' \right\} + \left\{ \frac{1}{2} (\psi^2 + \varphi^2) \right\} + \{ \psi \psi_t + \psi \psi' \}
\]

\[
e_{\varphi\theta}^{(p)} = e_{\varphi\theta}^{(1)} + e_{\varphi\theta}^{(2)} + e_{\varphi\theta}^{(p)} = \left\{ \frac{u}{r} + r \left( \frac{u'}{r} \right) \right\} + \{ \beta \psi + \psi \psi' \}
\]

\[
k_{\varphi}^{(p)} = \beta', \quad k_{\theta}^{(p)} = \psi - \frac{r'}{R_2}
\]

where

\[
\beta = \frac{u}{R_1} - u', \quad \psi = \frac{v}{R_2} - \psi', \quad \varphi = \frac{1}{2} \left( -\frac{\dot{u}}{r} + u' + \frac{ur'}{r} \right)
\]

Subscript "c" expresses the strain consisting of prebuckling and buckling deformation. Assuming \( \theta_m \) is the circumferential coordinate of the \( m \)-th meridional stiffener. \( Z_m \) is the distance of centroid of the stiffener area from shell midplane, letting \( \theta = \theta_m \) and substituting Eq. (2.1) and (2.2), we can obtain the strain relations of the \( m \)-th meridional stiffener:

\[
e_{m\varphi} = e_{m\varphi}^{(1)} + Z_m k_{m\varphi}, \quad e_{m\theta} = e_{m\theta}^{(1)} + Z_m k_{m\theta}
\]

where.

\[
e_{m\varphi} = e_{\varphi}^{(1)} + e_{\varphi}^{(2)} + e_{\varphi}^{(p)}, \quad e_{m\theta} = e_{\theta}^{(1)} + e_{\theta}^{(2)} + e_{\theta}^{(p)}, \quad k_{m\varphi} = k_{\varphi}, \quad k_{m\theta} = k_{\theta}
\]

In a similar manner, the strain relations of \( \epsilon \)-th circumferential stiffener: