INTERACTION OF SOUNDING ELECTROMAGNETIC WAVES WITH ELONGATED DEFECTS OF THE MATERIAL

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We develop a constructive method for the investigation of the vector interaction of electromagnetic waves with bodies. As an example, we consider a system of perfectly conducting cylindrical surfaces (screens) and a dielectric cylinder. The arrangement of scatterers and their cross sections are regarded as arbitrary. Problems of this sort are typical of the nondestructive electromagnetic inspection of elongated defects. They are solved by the method of singular integral equations. Integrals with logarithmic singularities, Cauchy-type kernels, and hypersingular integrals (in the sense of Hadamard) were found by using interpolational quadrature formulas. Numerical solutions of integral equations were obtained by the method of mechanical quadratures. We also present some examples of the calculation of the characteristics of diffraction fields in the far-field zone.

For the contemporary analysis of the durability of structural elements performed by the methods of fracture mechanics, it is necessary to have information about their defects. The required information is often obtained by applying electromagnetic methods of nondestructive inspection based on the theory of diffraction. In view of the facts that sounding can be performed within a broad frequency band and that the geometry of defects and their arrangement are arbitrary, we must focus our attention on the numerical simulation of the interaction of waves with macroscopic heterogeneities of the material. In order to solve scalar problems of diffraction theory which simulate the interaction of sounding waves with defects, one may use the methods developed in [1, 2]. Note that the numerical solution of the scalar problem of diffraction for an arbitrary system of cylindrical screens simulating thin rigid inclusions in the material located near the dielectric cylinder was given in [3, 4]. In this case, sounding wave is perpendicular to the collinear axes of cylindrical scatterers and the original problem reduces to the solution of two independent problems corresponding to E-polarized and H-polarized waves. At the same time, in the case of oblique waves, E-polarized and H-polarized waves interact in the dielectric material and the initial problem becomes much more complicated.

The aim of the present work is to generalize the method of singular integral equations developed in [1, 2, 5] to the solution of the vector problem of diffraction on a system of cylindrical screens with dielectric inclusions. As a result, we obtain the rigorous theory of electromagnetic interaction between arbitrary elongated defects irradiated by monochromatic waves.

Mathematical Statement of the Problem

Consider \( n \) open cylindrical screens and a homogeneous dielectric cylinder (bar) placed in a homogeneous isotropic material with relative electric permittivity \( \varepsilon_1 \) and magnetic permittivity \( \mu_1 \). Their generatrices are assumed to be infinitely long and parallel to the \( z \)-axis of the Cartesian coordinate system \( Oxyz \) (Fig. 1). If the material of the cylinder is characterized by the parameters \( \varepsilon_2 \) and \( \mu_2 \), then the wave numbers \( k_{1,2} \) are given by the formula

\[
k_{1,2} = \omega \sqrt{\varepsilon_1 \mu_1 \varepsilon_2 \mu_2},
\]

where \( \omega \) is a cyclic frequency and \( \varepsilon_0 \) and \( \mu_0 \) are the electric and magnetic permittivity of vacuum, respectively. The intersections of the cylinder and screens with the plane \( Oxy \) are described by arbitrary smooth contours \( L \) and \( L_v \) (\( L_v = a_v b_v, v = 1, N \)). By \( \Sigma_2 \) we denote the region bounded by the curve \( L \) and by \( \Sigma_1 \) we denote the exterior of the region \( \Sigma_2 \) in the plane \( Oxy \). In what follows, the quantities related to the regions \( \Sigma_1 \) and \( \Sigma_2 \) are equipped with the subscripts 1 and 2, respectively. We assume that, in tracing the curves \( a_v b_v \), the


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positive direction corresponds to the motion from \( a_v \) to \( b_v \) and, in tracing the contour \( L \), the counterclockwise direction is positive. In addition, the normals \( \vec{n}_v \) and \( \vec{n} \) directed to the left with respect to the positive direction of tracing the contours \( L_v \) and \( L \), respectively, are regarded as positive.

![Cross section of the collection of defects in the material.](image)

**Fig. 1.** Cross section of the collection of defects in the material.

Suppose that the components of a **TM-type** (TE-type) source electromagnetic wave are described by the following relations:

\[
\vec{W}_0(x, y, z) = \vec{W}_0^*(x, y) \exp(i\delta_0 z), \quad \vec{W}_0(x, y, z) = \begin{bmatrix} \vec{E}_0(x, y, z) \\ \vec{H}_0(x, y, z) \end{bmatrix}.
\]

It is necessary to determine the distribution of the total diffraction field formed as a result of the interaction of the source wave with defects. This problem is reduced to the solution of the vector Maxwell equations with the conditions of continuity of the tangential components of the field on the contours \( L_v \) and \( L \), namely,

\[
[\vec{n}_v, \vec{E}_t]_{L_v} = 0, \quad [\vec{n}_v, (\vec{E}_2 - \vec{E}_1)]_{L_v} = 0, \quad \text{and} \quad [\vec{n}, (\vec{H}_2 - \vec{H}_1)]_{L} = 0.
\]

To make the problem uniquely solvable, we also require that its solutions must satisfy the Sommerfeld conditions at infinity and the Meixner-type conditions near the edges of the screens.

**Integral Equations of the Problem**

Since the structure under consideration is homogeneous along the \( z \)-axis, we assume that the total diffraction field depends on the coordinate \( z \) in exactly the same way as the source wave (1). We represent the total field as a superposition of the source wave and the fields scattered by the dielectric inclusion and screens. To construct integral equations of the problem, we introduce scalar functions \( u_{1,2} \) and \( v_{1,2} \) which are, in fact, the longitudinal components of the total diffraction field, i.e., \( u_{1,2} \equiv E_{z(1,2)}(x, y) \) and \( v_{1,2} \equiv H_{z(1,2)}(x, y) \), and satisfy the Helmholtz equation.