THE STATE OF STRESS IN A MASS OF LOOSE ROCK IN THE CAVING ZONE DURING WORKING OF A STEEPLY DIPPING DEPOSIT

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During working of thick steeply dipping deposits with unstable rock in the hanging wall, the loose mass of rock in the caving zone is constantly undercut, and the level AB of mining operations descends (Fig. 1).

Let us consider the plane problem in polar coordinates (see Fig. 1). We will assume that the hanging-wall rock contacts the caving zone ABCD along a straight line AD. At the same time we will assume that the loose medium consists of homogeneous particles which are very small in comparison with the region under consideration.

The differential equations of equilibrium (see Fig. 1) are

\[ \frac{\partial \tau_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_\theta}{\partial \theta} + \frac{\tau_r - \tau_\theta}{r} + \gamma \sin \theta = 0; \]  
\[ \frac{\partial \tau_\theta}{\partial r} + \frac{1}{r} \frac{\partial \tau_\theta}{\partial \theta} + \frac{2\tau_\theta}{r} + \gamma \cos \theta = 0, \]

where \( \sigma_r \) and \( \sigma_\theta \) are the components of the normal stresses in the radial and tangential directions respectively; \( \tau_\theta \) is the tangential stress; \( \gamma \) is the density of the loose material; and \( r, \theta \) are the coordinates.

As a third and final equation we will take the equation of limiting stress state [1]:

\[ (\sigma_r - \sigma_\theta)^2 + 4\tau_\theta^2 = \sin^2 \varphi (\sigma_r + \sigma_\theta)^2, \]

where \( \varphi \) is the angle of internal friction of the loose material.

Thus we have a closed system of equations and boundary conditions in the planes AD and BC (see Fig. 1): when \( \theta = \theta_1, \tau_\theta = f_1\sigma_\theta \), and when \( \theta = \theta_2, \tau_\theta = f_2\sigma_\theta \), where \( f_1 \) and \( f_2 \) are the coefficients of friction of the loose material on the rocks of the foot and hanging walls of the deposit, respectively.

Let us now find an asymptotic solution of the problem [2].

The asymptotic stresses \( \sigma_r, \sigma_\theta, \) and \( \tau_\theta \) will be functions \( \sigma_r = \Phi_1(\theta), \sigma_\theta = \Phi_2(\theta), \tau_\theta = \Phi_3(\theta) \). In this case equations (1) and (2) with partial derivatives are transformed to ordinary linear differential equations with inhomogeneous terms, the solution of which is:

\[ \tau_\theta = -2\gamma r \cos \theta, \]

and on applying the boundary conditions we find that

\[ \sigma_r = -\frac{2\tau_\theta}{f_1 + f_2} (\cos \theta_1 - \cos \theta_2 + 1,5 f_1 \sin \theta_1 + 1,5 f_2 \sin \theta_2). \]

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Solving (3) for $\sigma_\theta$, we find that

$$\sigma_\theta = \sigma_r \cos^2 \varphi \left( 1 + \sin^2 \varphi \pm 2\sin \varphi \sqrt{1 - \cot^2 \varphi \frac{\sigma_\theta^2}{\sigma_r^2}} \right). \quad (6)$$

We substitute for $\tau_{r0}$ from (4) and for $\sigma_r$ from (5) under the radical; after some simple transformations we obtain the asymptotic relation between the normal radial and normal tangential stresses at any point in the loose medium:

$$\sigma_\theta = \sigma_r \psi(\theta), \quad (7)$$

where $\psi(\theta)$ is a functional factor representing the coefficient of lateral pressure, given by

$$\psi(\theta) = \frac{1}{\cos^2 \varphi} (1 + \sin^2 \varphi \pm 2\sin \varphi \sqrt{1 - K^2 \cos^2 \theta}), \quad (8)$$

where, in turn,

$$K = \frac{f_1 + f_2}{(\cos \theta_1 - \cos \theta_2 + 1.5f_1 \sin \theta_1 + 1.5f_2 \sin \theta_2) \tan \varphi}. \quad (9)$$

From (8) it follows that the coefficient of lateral pressure can take on a maximum or minimum value depending the acting pressure [3],

$$\frac{\sigma_\theta}{\sigma_r} = \psi(\theta) \frac{\text{min}}{\text{max}}.$$

The minimum value of the coefficient of lateral pressure corresponds to the expression

$$\psi(\theta) = \frac{1}{\cos^2 \varphi} (1 + \sin^2 \varphi - 2\sin \varphi \sqrt{1 - K^2 \cos^2 \theta}), \quad (10)$$

which shows that the crucial stress (the acting pressure) is the normal radial stress $\sigma_r$. In this case the loose medium is in its minimal (active) state of stress.

In the working of thick steeply-dipping deposits with unstable hanging-wall rocks, the rock pressure is due to the weight of the loose rock mass in the zone of caving.

It is interesting to note that when $f_1 = f_2 = 0$ and also when $\theta = 90^\circ$, independently of the values of $f_1$ and $f_2$ the functional factor can be expressed in the form

$$\psi(\theta) = \frac{1 - \sin \varphi}{1 + \sin \varphi},$$

i.e., it is a constant coefficient of lateral pressure which depends on $\varphi$ only.

We must derive the equation of equilibrium for an elementary ring sector with dimensions $\alpha \, dr$ (Fig. 2) to solve the general problem.