MEASURES ON DIFFEOMORPHISM GROUPS FOR NON-ARCHIMEDEAN MANIFOLDS: GROUP REPRESENTATIONS AND THEIR APPLICATIONS

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Nondegenerate σ-additive measures with ranges in R and Q_q (q ≠ p are prime numbers) that are quasi-invariant and pseudodifferentiable with respect to dense subgroups $G'$ are constructed on diffeomorphism and homeomorphism groups $G$ for separable non-Archimedean Banach manifolds $M$ over a local field $K$, $K \supset Q_p$, where $Q_p$ is the field of $p$-adic numbers. These measures and the associated irreducible representations are used in the non-Archimedean gravitation theory.

1. Introduction

Diffeomorphism groups and their irreducible representations are important in quantum mechanics and quantum gravitation theory [1]. Recently, the non-Archimedean quantum mechanics and quantum gravitation theory have also been developed [2, 3].

Sections 2 and 3 of this paper are devoted to constructing measures. Section 3.7 relates to studying the irreducibility of regular unitary representations for the diffeomorphism group $G'$. Section 4 presents examples of these representations in the non-Archimedean quantum mechanics and quantum gravitation theory.

2. Specific isomorphisms of spaces

2.1. Let $C(\alpha_r, M \rightarrow Y)$ denote the space of analytic maps $g: M \rightarrow Y$ with a radius of convergence not less than $r$, $0 < r < \infty$, where $M$ is clopen in $B(X, 0, r)$, $X$ and $Y$ are Banach spaces (BS) over $K$, and $B(E, x, R) := \{y \in E : d(y, x) \leq R\}$ is a ball in the metric space $(E, d)$.

2.2.1. We take a sequence of BS $\{H(n) : n \in \mathbb{N}_0\}$ over the field $K$, where $\mathbb{N}_0 := \{0, 1, 2, \ldots\}$ and $\mathbb{N} := \{1, 2, \ldots\}$, and define the BS

$$c_0(H(n) : n) := \{f = (f_0, f_1, f_2, \ldots) : f_n \in H(n) \text{ for any } n \text{ and } \|f^n\|_{H(n)} \rightarrow 0 \text{ as } n \rightarrow \infty\}$$

with the norm

$$\|f\|_{c_0(H(n) : n)} := \sup_n \|f^n\|_{H(n)}.$$

Let $X$ be a separable-type BS over $K$. Then it is isomorphic to $c_0(\alpha, K) := c_0(K : n)$ for $\alpha = \omega_0$ and to $c_0(\alpha, \mathbb{K}) = \mathbb{K}^\alpha$ for $\alpha < \omega_0$, where $\alpha$ is an ordinal, $\alpha \leq \omega_0$, $\omega_0$ is the initial ordinal of power $\aleph_0 := \text{card}(\mathbb{N})$, $\dim_K X := \text{card}(\alpha)$, and the BS $X$ has the standard orthonormal basis $(e_j : j \in \alpha)$ [4].

Here and henceforth, we consider analytic Banach manifolds (BM) $M$ and $N$ modeled on $X$ and $Y$ with atlases $At(M)$ and $At(N)$ consisting of disjunctive charts $(U_j, \phi_j)$, $j \in \Lambda$, and $(V_j, \psi_j)$, $j \in \Omega$, $\Omega \cup \Lambda \subset \mathbb{N}$, where $U_j, \phi_j(U_j), V_j,$ and $\psi_j(V_j)$ are clopen in $M, X, N,$ and $Y$ respectively, $\phi_j: U_j \rightarrow \phi_j(U_j)$ and $\psi_j: V_j \rightarrow \psi_j(V_j)$ are homeomorphisms, and $\phi_j(U_j) = B(X, x_j, r_j), 0 < r_j < \infty,$ for any $j$.
2.2.2. For \( \Lambda = \omega_0 \), we define the BS

\[
C_\ast(t, M \to Y) := \left\{ f \mid U_j \in C_\ast(t, U_j \to Y) : \|f\|_{C_\ast(t, M \to Y)} := \sup_{j \in \Lambda} \frac{\|f|_{U_j}\|_{C_\ast(t, U_j \to Y)}}{\min(1, r_j)} < \infty \right\}
\]

where \( 0 \leq t < \infty, * = 0 \) for \( C_0(t, U \to Y) \), and * = \( \emptyset \) or * is simply dropped for \( C(t, U \to Y) \) (for \( C(t, M \to Y) \)), in the case of a finite \( \text{At}(M) \), also see Sec. 2 in [5] and [6]). Let \( C_\ast^0(t, M \to N) \), \( 0 \leq t < \infty \), denote the space of functions \( f : M \to N \) such that \( (f_t - \theta_i) \in C_\ast(t, M \to Y) \) for any \( i \in \Omega \), \( f_t = \psi_i \circ f \), and \( \theta_i = \psi_i \circ \theta \). We similarly define \( C_\ast((t, s); M \to N) \) (see [7]) for \( \dim K = n \in \mathbb{N} \). We introduce the groups

\[
G(\tau, M) := C_\ast^0(\tau, M \to M) \cap \text{Hom}(M)
\]
called diffeomorphism groups (and homeomorphism groups for \( 0 \leq t < 1 \) and \( s = 0 \)), where \( \text{Hom}(M) \) is the group of continuous homeomorphisms, \( \tau = (t, s) \) or \( \tau = \text{an}_{t,s} \), and \( s = 0 \) is dropped.

2.3. We set \( \mathcal{B} := \{ x = (x^i : i \in \alpha) \in B : \text{there is an } i \in \alpha \text{ with } x^i = 0 \text{ or } x^i = 1 \} \) for \( B(X, 0, 1) =: B \).

For each \( \tau \in \Gamma_K := \{ \tau \in K : 0 \neq \tau \in K \}, \infty > \tau > 0 \), we fix \( \xi_i \in \mathbb{K} \) with \( |\xi_i|_K = \tau \), define \( \mathcal{B}(X, y, \tau) := y_i + \mathcal{B}(X, 0, 1) \times \xi_i \), where \( \{ y_i : i \in \mathbb{N} \} \) is a fixed chosen set for \( \tau \) such that \( y_1 = 0 \) and \( X \) is a disjunctive union of balls in \( \mathcal{B}(X, y, \tau) \), and set

\[
\mathcal{M} := \bigcup_j \phi_j^{-1}(\mathcal{B}_j)
\]

(see [5] and Sec. 2.2.1).

2.4. Let \( u > p \) and \( s > 1 \) be integers. Then the algebraic linear system of equations

\[
c(d, d, w)x(d, u) + \cdots + c(p, d, w)x(p, u) = 0 \quad (d = 0, \ldots, w - 1, \ w = 1, \ldots, \min(s - 1, u) =: p)
\]

\[
\chi(0, u) + \cdots + \chi(p, u) = 1
\]

where

\[
c(j, d, w) := \frac{(u - j)!j!}{(w - d)!(u - j - w + d)!(j - d)!(d + 1)!}
\]

has a unique solution with \( \chi(j, u) \in \mathbb{Q} \) (see [8, 9]).