A theorem is given which provides a bound on the departure, at any point in a spatially distributed reaction system, of the total concentration of a conserved moiety from the volume mean. An immediate consequence is that the total concentration of an enzyme in an open reaction system may be considered as (spatially) constant, wherever the diffusion coefficients of the enzyme and its complexes are sufficiently similar.

Introduction. In this paper we consider the spatial distribution of a conserved chemical species (or group) in an open reaction-diffusion system. As an example we may take an enzyme and its various bound forms. Suppose the enzyme to be diffusible and confined to a cell the surface (membrane) of which is impermeable to both bound and free enzyme. Then the total number of enzyme molecules (free and bound) is fixed. We inquire as to the spatial distribution of the enzyme containing species in the steady state and in particular the distribution of total concentration of enzyme containing species.

This is of interest theoretically since local rates of reaction are determined, in part, by the local concentration of spatially distributed catalysts. It is of interest practically because of simplifications which can sometimes be made as a result of such considerations. As an example of the latter, consider a substrate which is converted by a simple Michaelis-Menten enzyme (or indeed, by any
ordered sequence pathway catalyzed by a single enzyme). In the diffusion equation for that substrate, \( D \nabla^2 C - Q = 0 \), we are tempted intuitively to replace \( Q \), the local rate of consumption, by the standard Michaelis–Menten–Haldane type expression: \( Q = \frac{V_m C}{K_m + C} \). Now, in general, this is not correct when the enzyme is diffusible. The extent to which this is a serviceable approximation depends upon consideration the formal basis of which is given in what follows. Whenever the approximation is acceptable, we gain tremendous simplicity in that we then deal with a single equation as opposed to a set, of minimum number three, with accompanying boundary conditions.

Some Preliminaries. We consider a region \( R \) of volume \( V \) in which \( N \) chemical species diffuse and react. It is assumed that the simple Fick Law of diffusion is obeyed and that all diffusion coefficients are constant. If \( C_i \), \( D_i \) and \( Q_i \) are the concentration, diffusion coefficient and rate (per unit volume) of production of the \( i \)th species, we have at each point of \( R \) in the steady state
\[
D_i \nabla^2 C_i + Q_i = 0, \quad 1 \leq i \leq N.
\]
It is a simple matter of stoichiometry that in general there will be at least one subset* of species such that \( \sum \alpha_k Q_k = 0 \), where \( \alpha_k > 0 \) and the summation is over the subset. This results basically from the fact that in the chemical reactions occurring among the species there is always conservation of atomic types and usually conservation of certain groups and/or molecules. This has been discussed in detail elsewhere (Hearon, 1965), examples given and the relation of the \( \alpha_k \) to the stoichiometric coefficients of the chemical reactions set down in detail.

We will focus attention on a particular conserved subset and it is no restriction to assume the species to be numbered so that for some \( 2 < n < N \) and \( \alpha_i > 0 \), we have
\[
\sum_{i=1}^{n} \alpha_i Q_i = 0. \tag{2}
\]
It follows from (1) and (2) that the function
\[
\phi = \sum_{i=1}^{n} \alpha_i D_i C_i, \tag{3}
\]
obeys \( \nabla^2 \phi = 0 \) in \( R \) and is thus a harmonic function in \( R \). Let
\[
f = \sum_{i=1}^{n} \alpha_i C_i, \tag{4}
\]
* If not all of the reactions are monomolecular, there will be at least one proper subset.