The Bayes Estimator in a Misspecified Linear Regression Model

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SUMMARY

The paper presents an optimal Bayes estimator in the linear regression model under misspecification. This estimator is compared with the ordinary least squares estimator in terms of the matrix mean square error criterion. Some results on admissibility and prediction are also derived.

Keywords: LINEAR REGRESSION MODEL; MISSPECIFICATION; OPTIMAL BAYES ESTIMATOR; ADMISSIBILITY; PREDICTION.

1. INTRODUCTION

Let the true regression model be given by

\[ y = Q\gamma + \varepsilon = X_1 \beta_1 + X_2 \beta_2 + \varepsilon, \tag{1.1} \]

where \( y \) is an \( n \times 1 \) vector of observations on the dependent variable, \( X_1 \) and \( X_2 \) are \( n \times k_1 \) and \( n \times k_2 \) matrices, \( \beta_1 \) and \( \beta_2 \) are \( k_1 \times 1 \) and \( k_2 \times 1 \) vectors with \( k_1 + k_2 = k \). \( Q = (X_1, X_2) \) is an \( n \times k \) matrix of nonstochastic regression observations of rank \( k \), \( \gamma = (\beta_1', \beta_2')' \) is a \( k \times 1 \) vector of unknown random coefficients, and \( \varepsilon \) is an \( n \times 1 \) vector of unknown disturbances such that \( E(\varepsilon) = 0 \) and \( \text{cov}(\varepsilon) = \sigma^2 I \) with \( \sigma^2 > 0 \) known. Without loss of generality, assume that \( Q'Q = I \),
i.e. $Q$ is a column orthonormal matrix, otherwise there would exist a $k \times k$ upper triangular matrix $T$ and an $n \times k$ matrix $P$ with its columns orthonormal such that $Q = PT$, i.e. (1.1) becomes

$$y = Q\gamma + \epsilon = PT\gamma + \epsilon = P\gamma^* + \epsilon,$$

where $\gamma^* = T\gamma$ and $P'P = I$. By the property $Q'Q = I$ it is obvious that $X'_1X_2 = 0$ and $X'_2X_1 = 0$.

Suppose the model (1.1) is misspecified as

$$y = X_1\beta_1 + u,$$

(1.2)

where $u = \delta + \epsilon$ with $\delta = X_2\beta_2$. Furthermore let the prior distribution of $\beta_1$ satisfy the following moment conditions:

$$E(\beta_1) = \beta_0, \quad \text{cov}(\beta_1) = \sigma^2\Sigma_0,$$

(1.3)

where $\beta_0$ and $\Sigma_0$ are given $k_1 \times 1$ and $k_1 \times k_1$ matrices, and $\Sigma_0$ is positive definite (p.d.).

The ordinary least squares estimator (LSE) of $\beta_1$ in model (1.2) is

$$\hat{\beta}_1 = (X'_1X_1)^{-1}X'_1y.$$  

(1.4)

There are three different approaches concerned with the Bayes Estimator (BE) in the linear model if the prior distribution is partly unknown. In the first approach, the given prior distribution is a normal distribution with hyperparameters, and usually the proposed BE is a function of hyperparameters (see Lindley and Smith (1972)). In the second approach the prior distribution of the parameter is unknown, e.g. only the moment conditions of prior distribution are given. Hence the BE may be written in terms of the marginal density of a sufficient statistic (see Clemmer and Krutchkoff (1968)). Usually these first two approaches are related to the Empirical Bayes Estimator (EBE). For the first approach, one can get the EBE by using the sample to estimate the hyperparameters (see Efron and Morris (1972), Berger (1985) and Ghosh et al. (1989)). For the second approach, one can obtain the EBE by using the estimation of density and its derivatives (see Clemmer and Krutchkoff (1968), Berger (1985) and Singh (1985)). The third approach yields the Linear Bayes Estimator (LBE). In this case only some moment conditions of the prior