Extensions of Dinkelbach’s Algorithm for Solving Non-linear Fractional Programming Problems

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Abstract

Dinkelbach’s algorithm was developed to solve convex fractional programming. This method achieves the optimal solution of the optimisation problem by means of solving a sequence of non-linear convex programming subproblems defined by a parameter.

In this paper it is shown that Dinkelbach’s algorithm can be used to solve general fractional programming. The applicability of the algorithm will depend on the possibility of solving the subproblems.

Dinkelbach’s extended algorithm is a framework to describe several algorithms which have been proposed to solve linear fractional programming, integer linear fractional programming, convex fractional programming and to generate new algorithms. The applicability of new cases as nondifferentiable fractional programming and quadratic fractional programming has been studied.

We have proposed two modifications to improve the speed-up of Dinkelbach’s algorithm. One is to use interpolation formulae to update the parameter which defined the subproblem and another truncates the solution of the subproblem. We give sufficient conditions for the convergence of these modifications.

Computational experiments in linear fractional programming, integer linear fractional programming and non-linear fractional programming to evaluate the efficiency of these methods have been carried out.

Key Words: Fractional Programming, Parametric Optimization, Nonconvex Programming, Nonlinear Programming.

AMS subject classification: 90C32, 90C31, 90C26, 90C30

1 Introduction.

The non-linear fractional programming problem, i.e., the minimisation of a fraction of two functions subject to given conditions, arises in various decision making situations; for example linear fractional programming is used

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in fields of game theory [Isbell and Marlow (1956)], network flows [Arisawa and Elmaghraby (1972)]; the quadratic fractional programming problem is used in field production planning and inventories [Swarup (1965)]; the integer linear fractional programming problems find their applications in quite a large number of areas such as fixed-charge problems, job-shop scheduling problems, traffic planning [Spiess and Florian (1989)], etc. A review of various applications is given by Schaible (1982), which covers more than 550 articles.

There are different solution algorithms for determining the optimal solution of particular kinds of fractional programming problems. For example, the authors Charnes and Cooper (1962), Isbell and Marlow (1962), Martos (1964) and Wolf (1985) solve linear fractional programming. Integer linear fractional programming has been solved by Rajendra (1993), Seshan and Tibekar (1980), Chandra and Chandramoham (1980), etc. Swarup (1965) gives an algorithm for solving quadratic fractional programming. The case where the restrictions are linear and the objective function is the quotient of a convex function with a concave function is solved by Mangasarian (1969) using Frank and Wolfe's algorithm (1956). Dinkelbach (1968) also considered the same objective over a convex feasible set. He solved this problem by means of the solution of a sequence of non-linear convex programming problems.

There are other fields of application where exact algorithms do not exist to solve fractional programming. An example of this is given in Gopal et al. (1991) which investigates configuration management and optimal logical network design for reconfigurable networks. They defined underlying constrained non-linear integer fractional problems and developed a heuristic technique to solve it.

In this paper several algorithms are presented to solve non-linear fractional programming based on the parametric approach to the fractional programming problem given by Dinkelbach. We prove their convergence and this opens the possibility of developing new exact algorithms or heuristic procedures for problems formulated by means of fractional programming.

The remainder of the paper is organised as follows. In Section 2 we introduce the fractional problem and Dinkelbach's algorithm and we prove the global convergence of Dinkelbach's algorithm for the general case of the fractional programming. In sections 3 and 4 two possible extensions are given for the speed-up of Dinkelbach's algorithm. The first consists