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Queuing Analysis of a Sequential Recognizer of Patterns

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Abstract

A recognition procedure is modelled as a queuing problem. We obtain first the steady-state distribution of probabilities of the basic process describing its evolution. Then, we derive some performance measures of the recognition procedure such as the waste time, the queue length of patterns, the classification error probabilities and so on. Stochastic comparisons are also provided. Our study extend the Viscolani's results to the non Markovian case.

Key Words: Pattern recognition, Performance evaluation, Queuing system, Stationary distribution, Classification error probabilities, Stochastic comparison.

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1 Introduction

We consider the recognition problem of patterns defined by a given set of patterns $\mathcal{R} = \{\chi\}$ and a partition $\{D_1, D_2, ..., D_k\}$ of $\mathcal{R}$, indicating the $k$ possible classifications of each pattern $\chi \in \mathcal{R}$. A classification rule is an application $d(.)$ which to each pattern $\chi \in \mathcal{R}$ links a classification $d(\chi)$ in the set $\{1, 2, ..., k\}$. The classification $d(\chi)$ of the pattern $\chi$ is said to be correct if $d(\chi) = i$ and $\chi \in D_i$; otherwise it is said to be wrong.

Let $\chi_1, \chi_2, ..., \chi_n, ...$ be the sequence of the patterns we have to recognize, and let $t_1 < t_2 < ... < t_n < t_{n+1} < ...$, be the ordered times of their arrivals, where $t_n$ is the time of arrival of the pattern $\chi_n$. We assume that the counting process of patterns $N(t) = \max\{n : t_n \leq t\}$ follows an homogeneous Poisson process at rate $\lambda > 0$.

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The classification of each pattern is realised by a sequential recognizer similar to that of SPRT (Sequential Probability Ratio Test) described by Fu K.S. (1968). There is a device ∑ which links a sequence of classifications \( d_1(\chi_n), d_2(\chi_n), ..., d_m(\chi_n) \) to every pattern \( \chi_n \). Each decision concerning the pattern \( \chi_n \) is based on the information carried by the first \( m \) features of this pattern. This approach is different from those which are used in traditional recognizers. The latter exploits all the information carried by the pattern’s features in order to perform the classification. This approach allows us to take into account the cost involving by the measures collection. It is also useful if in the recognition problem the descriptive features of patterns are naturally sequential.

Let \( r_{m,n} \) be the time taken by the recognizer ∑ to perform the classification \( d_m(\chi_n) \) (\( m, n \geq 1 \)). The sequence \( \{r_{m,n}\} \) forms a sequence of independent and identically distributed random variables with common distribution function \( H(x) = P\{\tau < x\}, \( H(0^+) = 0 \), with Laplace-Stieltjes transform \( h(s) = E\{e^{-st}\} \) (Re\( (s) \geq 0 \)) and with \( k \)th order moment \( \tau^k = E\{\tau^k\} \) \((k \geq 1)\). Here, \( \tau \) denotes the stationary service time and \( E\{\cdot\} \) denotes the mathematical expectation relative to the basic probability space \( (\Omega, \mathcal{I}, \mathcal{P}) \). The sequences \( \{r_{m,n}\} \) and \( \{t_n\} \) are assumed to be independent.

Assume that a pattern \( \chi_n \) arrives at the system at time \( t_n, n \geq 1 \). If the recognizer ∑ is idle at this time, then it starts computing the classification, \( \{d_m(\chi_n), i = 1, 2, ..., m\} \) of the pattern \( \chi_n \) just arrived. After computation of \( d_m(\chi_n) \) (for \( m \geq 1 \)) the recognizer starts a new classification \( d_{m+1}(\chi_n) \) of the same pattern \( \chi_n \) with probability \( \alpha_m \) or stops the actual classification sequence with probability \( 1 - \alpha_m \). In the latter case, the recognizer ∑ is ready to begin the classification sequence for any possible new pattern.

The recognizer ∑ may be interrupted at every instant during its operation, except when it is computing the first classification \( d_1 \). If ∑ is computing \( d_m(\chi_n) \) and a new pattern \( \chi_{n+1} \) arrives, then it interrupts that operation and starts computing the first classification, \( d_1(\chi_{n+1}) \), of the pattern just arrived according to the rule described above. If the new arrived pattern \( \chi_{n+1} \) find ∑ busy and the queue is non-empty, then it joins the last place in the queue. The service discipline in the queue is first-in-first-out. Note that the number of features of each pattern is variable and may be bounded above by positive integer, say \( M \). In this case, we have \( \alpha_M = 0 \) and any classification sequence has a length \( \leq M \). In the present paper, we restrict ourselves to the case when \( M = \infty \). A detailed discussion of