ROTATIONAL HYPERSURFACES OF SPACE FORMS
WITH CONSTANT SCALAR CURVATURE

Maria Luiza Leite *

Let $M$ be a complete rotational hypersurface of a space form with constant scalar curvature $S$. In this paper we classify these hypersurfaces in the cases of $\mathbb{R}^n$ and $\mathbb{H}^n$, determine the admissible values of $S$ in each of the three spaces and give a geometrical description of the hypersurfaces according to the values of $S$. In the case of $\mathbb{S}^n$ we find examples of embedded hypersurfaces with constant $S \in (\frac{n-2}{n-1}, 1)$, which are not isometric to product of spheres.

The scalar curvature $S$ of a riemannian manifold is an important geometric invariant, thus the interest in those manifolds with constant $S$ and in particular, in the hypersurfaces of space forms.

One important result is the theorem of A. Ros [7] according to which the only embedded compact hypersurfaces of $\mathbb{R}^n$ with constant $S$ are round spheres. For the non-compact ones there is a theorem of Cheng-Yau [3] stating that the only complete examples with sectional curvatures $K \geq 0$ are $S^{k-1} \times \mathbb{R}^{n-k}$, $1 \leq k < n$.

In [4] and [5] Hsiang analysed rotational hypersurfaces of space forms with a symmetric function $\sigma_j$ of the principal curvatures constant, which includes constant scalar curvature when $j = 2$. There he obtains a collection of complete

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hypersurfaces of $\mathbb{R}^n$ and $\mathbb{H}^n$ with $S > 0$ and of $S^n$ with $S > 1$, but no classification theorem is presented.

In this paper we classify all complete rotational hypersurfaces of $\mathbb{R}^n$ and $\mathbb{H}^n$ with constant scalar curvature (Theorems 3.4 and 3.5). Partial results for $S^n$ are presented in Theorem 3.6. We also prove that $S$ is precisely greater than or equal to the space form curvature, except in the case of $S^n$ where any value greater than $(n - 3)/(n - 1)$ is admissible. In particular we exhibit a collection of new complete hypersurfaces of $\mathbb{H}^n$ with $S$ ranging in $[-1,0]$, of $\mathbb{R}^n$ with $S = 0$ and of $S^n$ with $S$ in the interval $(\frac{n-3}{n-1},1)$. Surprising examples of embedded hypersurfaces of $S^n$ with $S < 1$ are presented. We point out that Theorem 3.4 has been announced earlier (see [2]).

Our results suggest interesting problems in Global Differential Geometry. We state below three of these: the first one carries a flavor of Hilbert theorem for surfaces and the second a flavor of Bernstein theorem for minimal surfaces.
- Is there a complete hypersurface of $\mathbb{R}^n$ with constant $S < 0$?
- Is there a nonflat complete graph in $\mathbb{R}^n$ with constant $S = 0$?
- Are there embedded hypersurfaces of $S^n$ with constant $S \geq 1$ other than product of spheres?

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1. UNIFIED EXPRESSION FOR THE SCALAR CURVATURE.

We denote by $N_c$ the simply connected $n$-dimensional space form of constant curvature $c = 0, 1$ or $-1$. We will take as models for $N_c$ the euclidean space $\mathbb{R}^n$, the unitary round sphere $S^n$ and the hyperbolic upper space $\mathbb{H}^n = \{y \in \mathbb{R}^n : y_n > 0\}$.

Let $M$ be a rotational hypersurface of $N_c$, that is, invariant by the orthogonal group $O(n - 1)$ considered as a subgroup of isometries of the ambient space. To study the geometry of $M$, we generalize the method used by Spivak ([8], page 173) to compute the intrinsic Gaussian curvature of a rotational surface of $\mathbb{H}^3$.

There, an element of $O(2)$ fixes all points of a given geodesic $\gamma$, which is the axis of revolution, and rotates the initial tangent vector of a geodesic ray starting orthogonally from $\gamma$. The orbit of a point $p$, at a distance $r > 0$ from $\gamma$, under the