A SIGNIFICANT EXAMPLE TO TEST METHODS FOR SOLVING SYSTEMS OF NONLINEAR EQUATIONS

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ABSTRACT - A system of nonlinear equations verifying certain conditions is considered. Some resolution methods are applied and the relevant results are displayed.

1. Introduction.

There exist many numerical methods for solving systems of nonlinear algebraic and/or transcendental equations, but up to now there are few model problems [1, 2, 3] to test them.

Therefore, we consider interesting a system of $n$ nonlinear equations, for which:

(i) $n$ can be as large as required;
(ii) a unique solution always exists;
(iii) an initial guess sufficiently close to the solution can be easily found.

2. The example.

2.1. Denoting by $\mathbf{X}$ and $\mathbf{F}(\mathbf{X})$ the vectors whose components are the variables $x_h$ and the functions $f_h(\mathbf{X})$ ($h = 1, 2, \ldots, n$) respectively, the system of nonlinear equations

\[ f_h(x_1, x_2, \ldots, x_n) = 0 \quad (h = 1, 2, \ldots, n) \]

can be written as

\[ \mathbf{F}(\mathbf{X}) = 0. \]
If the functions \( f_h(X) \) are differentiable in each point \( X \) of the real \( n \)-dimensional space \( S_n \) and if there exist numbers \( m_{hk} \) and \( p_h \) such that, for every \( X \) of \( S_n \),

\[
\left| \frac{\partial f_h(X)}{\partial x_k} \right| \leq m_{hk} \quad (h, k = 1, 2, \ldots, n)
\]

and

\[
\frac{\partial f_h(X)}{\partial x_h} \geq p_h > q_h \quad (h = 1, 2, \ldots, n)
\]

where \( q_h = \frac{1}{2} \sum_{k=1}^{n} (m_{hk} + m_{kh}) \), then \([4]\)

(2) \[
\| F'(X') - F'(X'') \|_2 \leq K \| X' - X'' \|_2
\]

and

(3) \[
(F(X') - F(X''), X' - X'') \geq c (X' - X'', X' - X'')
\]

hold for any pair \( X', X'' \) of \( S_n \), with

\[
K = \sqrt{\max_{1 \leq j \leq n} \sum_{h=1}^{n} m_{hj} \sum_{k=1}^{n} m_{hk}}
\]

and

\[
c = \min_{1 \leq h \leq n} (p_h - q_h).
\]

Furthermore \([4]\), when (2) and (3) hold, the system (1) admits a unique solution \( X^* \) of which a convenient approximation is

(4) \[
X = -\frac{F(0)}{2} \frac{c + K}{cK}.
\]

2.2. In order to test various methods for solving nonlinear systems, we propose the following example of (1)

(5) \[
f_h(X) = \sum_{k=1}^{n} \left[ x_k^2 + h/k \left( \sin^2 \log \left( x_k^2 + h/k \right) + \cos^2 \log \left( x_k^2 + h/k \right) \right) \right] +
\]

\[
+ \beta n x_h + \left( h - \frac{n}{2} \right) \gamma = 0 \quad (h = 1, 2, \ldots, n),
\]

where \( \alpha, \beta \) and \( \gamma \) are positive integers with \( \beta > \alpha > 1 \). The functions of