COMBINATORIAL PROBLEMS OVER POWER SETS

G. Ausiello, A. Marchetti-Spaccamela (¹), M. Protasi (²)

ABSTRACT - In this paper results concerning structural and approximability properties of NP-complete optimization problems are considered. First, various approaches to the definition of such concepts as structure of a combinatorial problem and structure preserving reductions are presented. A large class of optimization problems is then introduced. These problems may be formalized as optimization problems over a power set. For this class most of the above considered concepts may be neatly formulated and developed.

1. Introduction.

In the last years combinatorial mathematics has been extensively applied to very different areas of research. In particular one of the branches in which combinatorics has been more used is the theory of computational complexity. The study of combinatorial algorithms has received much more attention than in the past and both from a theoretical point of view and from a practical point of view much research effort has been devoted to this area. This seems very natural; in fact in many parts of mathematics and computer science combinatorial algorithms very often occur.

Even if at all levels of complexity combinatorial techniques may be successfully used, in this paper we will mainly concentrate our attention to study the relationship between combinatorial and computational properties of NP-complete problems. As it is known, the class of NP-complete problems is very large and its importance derives from the fact that there is a general agreement that

Received February 14, 1980.
(¹) CSSCCA - CNR Istituto di Automatica, Università di Roma.
(²) Istituto Matematico, Università dell'Aquila.
no \textit{NP}-complete problem can be solved by an efficient (at most of polynomial time complexity) algorithm.

In par. 2, we will very briefly survey different approaches to the study of \textit{NP}-complete optimization problems. Our study will not be exaustive, and instead of presenting the various approaches in every detail we have preferred to stress informally what are the basic ideas and results. The aim is to give an up to date view of the main attempts which have made for characterizing \textit{NP}-complete problems from a combinatorial point of view.

In par. 3 we define a new class of optimization problems (max subset problems) that, from one hand is sufficiently large to include many famous combinatorial problems but that, on the other hand, for its characteristics, seems suitable to reveal interesting properties for the classification of \textit{NP}-complete optimization problems. Therefore, we will study the properties of the class of max-subset problems trying to see what are the relationships with some of the approaches presented in par. 2.

Finally in par. 4, we will define, in our formalization, various types of reductions and, also in this case, we will make a comparison between the old and new definitions, always confining our efforts to the case of max subset problems.

In Appendix the definitions of some of the combinatorial problems referred throughout the paper are presented (In [3] a very broad list of \textit{NP}-complete problems is given).

2. Combinatorial structure of optimization problems.

The concept of polynomial reduction plays a central role in the field of \textit{NP}-completeness. In fact, exhibiting a reduction from a problem \textit{A}, which is already known to be \textit{NP}-complete, to a problem \textit{B} it is sufficient to show that \textit{B} is \textit{NP}-hard, that is as hard as the hardest problem solvable in polynomial time by a non deterministic Turing-machine. If \textit{B} is also known to be in the class \textit{NP}, then to exhibit the reduction from \textit{A} to \textit{B} coincides with proving the \textit{NP}-completeness of \textit{B}.

On the other hand the behaviour of a problem with respect to the possibility to have « good » approximation algorithms is generally studied looking at the characteristics of the problem in itself.

Therefore it is natural to think that a combinatorial approach devoted to deepen the knowledge of computational properties should try one of the two following possibilities: 1) to strengthen the concept of reduction 2) to discover some invariants of the combinatorial structure of the problem or better to follow both the ways.