THRESHOLD METHODS FOR THRESHOLD MODELS IN AGE DEPENDENT POPULATION DYNAMICS AND EPIDEMIOLOGY

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Abstract - We construct a first order numerical method over bounded domains which discretizes the McKendrick equations for the nonlinear dynamics of an age-structured population. Under suitable hypotheses, the solution of the continuous model stabilizes towards a stationary solution as time goes to infinity. Our numerical solution inherits a similar large time behaviour.

In a second step, we consider an epidemic model with an age-structure of the S.I.S. type. The main characteristic of the model is that the epidemic is spreading in the fluctuating population solution of the first model. We again construct first order methods for which the discrete solution and the continuous model have similar large time behaviours.

1. Presentation of the models

1.1. The model of Gurtin-McCamy. A stabilization result.

Let \( u(t, a) \geq 0 \) be the density of individuals of age \( a \) at time \( t \), such that:

\[
P(t) = \int_0^{+\infty} u(t,a) \, da \text{ is the total population.}
\]

The following nonlinear equations, introduced by M.E. Gurtin and R.C. McCamy in [5] and developed by other authors such as S. Busenberg and M. Jan-
nelli in [2] or G. Webb in [11] describe the evolution of an initial age-structured population \( u_0(a) \).

\[
\begin{align*}
\begin{cases}
{u_t} + u_a + \mu(a, P(t))u &= 0 & (a > 0, t > 0) \\
 u(t, 0) &= \int_0^{+\infty} \beta(a, P(t))u(t, a) \, da & (t > 0) \\
 P(t) &= \int_0^{+\infty} u(t, a) \, da & (t > 0)
\end{cases}
\end{align*}
\]

Here \( \mu(a, p) \) (resp. \( \beta(a, p) \)) is the age specific death rate (resp. fertility rate) when total population is at level \( p \).

We consider the following (HO) hypothesis to hold.

\[
\begin{align*}
(H0) \quad & u_0 \geq 0, \text{ continuous, with compact support.} \\
& \beta, \mu \in C^1. \\
& 0 \leq \beta(a, p) \leq \bar{\beta} < +\infty, 0 \leq \mu(a, p).
\end{align*}
\]

If \((H0)\) holds, M.E. Gurtin and R.C. McCamy have shown in [5] that problem (1) has a unique non negative solution, global in time.

Many authors put their interest on the asymptotic behaviour of the solutions of (1) in various situations (see for example [2], [5], [8], [11]). One needs to make some additional assumptions in order to observe simple behaviour: in this respect, we chose to collect \((H0)\) with the following hypothesis, referred to later on as \((H1)\).

\[
\begin{align*}
(H1) \quad & \frac{\partial_p \beta}{\beta} \leq 0. \\
& \mu = \mu_n(a, p) + \nu_c(p), \text{ with } \mu_n, \nu_c \in C^1, \frac{\partial_p \nu_c}{\nu_c} > 0, \frac{\partial_p \mu_n}{\mu_n} \geq 0. \\
& \text{There exists } A > 0 \text{ such that } \beta(a, p) = \mu_n(a, p) = 0 \text{ for } a \geq A. \\
& \text{supp}(\beta(., p)) \subset [0, A_2]. \exists A_1 \text{ s.t. } \beta(a, p) = \mu_n(a, p) > 0 \text{ for } a \in [A_1, A_2]. \\
& \lim_{p \to \infty} \nu_c(p) > \tilde{\beta}, \nu_c(0) = \alpha > 0.
\end{align*}
\]