Monotonicity Properties of the Nucleolus on the Domain of Veto Balanced Games

J. Arin
Departamento de Fundamentos de Análisis Económico I, University of the Basque Country
L. Aguirre etorbidea 83, 48015 Bilbao, Spain
Email: jeparugj@bs.ehu.es

V. Feltkamp
School of Management, PO Box 1203
6201 BE Maastricht, The Netherlands
Email: vincent.feltkamp@bigfoot.com.

Abstract
This note shows that the nucleolus does not satisfy aggregate monotonicity and strong monotonicity, even on the class of veto balanced games, while it does satisfy complementary antimonotonicity on this class.

Key Words: Nucleolus, veto balanced games.

AMS subject classification: 91A12.

1 Introduction

The nucleolus (Schmeidler (1969)) is a well known solution for transferable utility coalitional games (TU games) that satisfies interesting properties but fails to satisfy different basic properties of monotonicity. This is the case even if we deal with more restrictive domains such as convex games (Hokari (2000)). In this note we study this question in the domain of veto balanced games, games where one player has a veto control. Those games have been used to model interesting economic situations where a group of agents has a veto power (see Muto et al. (1988) and Muto et al. (1989).

1 This author thanks financial support provided by the project UPV 036.321-HA042/99 of Basque Country University and the project PB96-1469-C05-04 of the Ministry of Education and Science of Spain.

Our result is that the nucleolus satisfies the complementary antimonotonicity property but not the aggregate monotonicity property nor the strong monotonicity property. We will show that in the class of veto balanced games there exists a core solution that satisfies the three properties. Finally, we add a result concerning the class of convex games.

2 Preliminaries

A cooperative n-person game in characteristic function form is a pair \((N, v)\), where \(N\) is a finite set of \(n\) elements and \(v : 2^N \rightarrow \mathbb{R}\) is a real valued function on the family \(2^N\) of all subsets of \(N\) with \(v(\emptyset) = 0\). Elements of \(N\) are called players and the real valued function \(v\) the characteristic function of the game. Any subset \(S\) of the player set \(N\) is called a coalition. The number of players in a coalition \(S\) is denoted by \(|S|\). Given a set of players \(N\) and a coalition \(S \subseteq N\) we denote by \(S^c\) the set of players of \(N\) that are not in \(S\). Generally we shall identify the game \((N, v)\) by its characteristic function \(v\).

A distribution of \(v(N)\) among the players is represented by a real valued vector \(x \in \mathbb{R}^N\) where \(x_i\) is the payoff assigned by \(x\) to player \(i\). A distribution satisfying \(x_i \geq v(i)\) for all \(i \in N\) is called an imputation and the set of imputations is denoted by \(I(v)\). We denote \(\sum_{i \in S} x_i\) by \(x(S)\). The core of a game is the set of imputations that cannot be blocked by any coalition, i.e.

\[
C(v) = \{x \in I(v) : x(S) \geq v(S) \text{ for all } S \subseteq N\}.
\]

A game with a non-empty core is called a balanced game. A game \(v\) is a veto-rich game if it has at least one veto player and the set of imputations is non-empty. A player \(i\) is a veto player if \(v(S) = 0\) for all coalitions where player \(i\) is not present. A balanced game with at least one veto player is called a veto balanced game.

A solution \(\phi\) on a class of games \(\Gamma_0\) is a correspondence that associates with every game \((N, v)\) in \(\Gamma_0\) a set \(\phi(N, v)\) in \(\mathbb{R}^N\) such that \(x(N) \leq v(N)\) for all \(x \in \phi(N, v)\). If there is no confusion with the set of players we write \((v)\) instead of \((N, v)\). This solution is called efficient if this inequality holds with equality. The solution is called single-valued if for every game in the class the set contains a unique element.

Given a vector \(x \in \mathbb{R}^N\) the excess of a coalition \(S\) with respect to \(x\)