EXCITED-STATE POPULATIONS OF HYDROGENLIKE AND
HELIUMLIKE IONS AND DIAGNOSTICS OF DENSE TRANSIENT
PLASMAS FROM THE RELATIVE INTENSITIES OF SPECTRAL
LINES

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Abstract

A collisional-radiative model of H- and He-like ions is constructed. The model allows a determination of excited-state population densities for arbitrary Ne, Te, and ion composition. The coefficients \( r_0 \) and \( r_1 \), the collisional-radiative ionization and recombination coefficients for H-like ions and for the He-like ions F VIII and Mg XI are calculated. The methods of plasma diagnostics from the relative intensities of spectral lines are substantiated. It is indicated that the possibility of measuring Ne from the resonance-to-intercombination line intensity ratio of a [He] ion is determined by the knowledge of the ion composition and \( T_e \).

1. Introduction

This paper presents a calculation of excited-state (ES) populations in H- and He-like ions within the framework of a collisional-radiative model over a wide range of plasma parameters. The quasisteady approximation for ES populations is used. As is customary, in the low-density limit the ES populations in ion \( Z \) are derived in the coronal approximation, which includes the population from the ground states of ions \( Z \) and \( Z + 1 \) and cascade radiative transitions. Each event of level population is therewith followed by radiative recombination into the ground state, and stepwise processes in ionization are insignificant. With increasing \( N_e \), collisional transitions between the ESs and three-particle recombination into the ESs come into play, and the contribution of stepwise ionization increases. In the high-density limit, with the first excited level in the collisional range, collisional transitions prevail over radiative transitions, and the rate of stepwise ionization approaches the total rate of excitation from the ground state. In a stationary plasma, the ES populations approach the equilibrium (with respect to the ground state of the ion \( Z + 1 \)), or Saha-Boltzmann, populations:

\[
N_{Z+1}^F(n) = \frac{g_Z(n)}{2g_{Z+1}(1)} \left( \frac{2\pi\hbar^2}{m_e T_e} \right)^{3/2} N_e N_{Z+1} \exp \left( \frac{E_Z^*(n)}{T_e} \right),
\]

where \( T_e \) and \( N_e \) are the electron temperature and density, \( N_{Z+1} \) is the population of the ground state of the ion \( Z + 1 \), \( E_Z^*(n) \) is the ionization energy of level \( n \) in the \( Z \) ion, and \( g_Z(n) \) the statistical weight of the level.

The proper formulation of the problem requires that the highest excited ion level \( n_{\text{max}} \) included in the model be in the collisional range (which corresponds, e.g., to the condition \( N_e c(n_{\text{max}}, n_{\text{max}} + 1) > A(n_{\text{max}}) \)), where \( c \) and \( A(n) \) denote, respectively, the excitation rate and the total radiative decay rate, the condition specified being necessary but sometimes insufficient. In this case, the effective inclusion of levels with \( n > n_{\text{max}} \).
using an empirical technique (we assign the term energy gap, see Sec.3) may be helpful. If we are not dealing with an ultimately high-density plasma, the determination of the ES populations and the evaluation of the ionization and recombination coefficients require that sufficiently detailed collisional–radiative ion models be invoked, the total number of ESs included amounting up to several tens. Clearly the plasma diagnostics from relative intensities of spectral lines should rely on precisely such calculations in the context of a multilevel model. The use of simplified (e.g., three-level) models leads to numerically erroneous results and, in addition, qualitatively different effects drop out of consideration, such as the dependence of the resonance-to-intercombination line intensity ratio in a He-like ion on the ion composition. Attention is drawn to this dependence in Sec.5.

2. Quasi-Steady Approximation for ES Populations in Transient Plasmas

Generally, if we are to derive the population densities of \( n \) states, a set of \( n \) first-order differential equations must be solved: \( \frac{dN_z(n)}{dt} = S_z(n) \), where \( S_z(n) \) are the powers of the population sources. As an example, we specify the expression for the rate of population-density variation of a level \( n > 1 \) of ion [\( \text{H} \)]:

\[
S_H(n) = \sum_{n'}^{n_{\text{max}}} N_H(n') A_H(n', n) - N_H(n) A_H(n) + \\
+ N_e \left[ \sum_{n' \neq n} N_H(n') c_H(n', n) - N_H(n) \left( \sum_{n' \neq n} c_H(n, n') + c_H(n) \right) \right] + \\
+ N_{\text{nuc}} t_H(n) + N_e^2 t_{\text{nuc}} t_H(n),
\]

where the coefficients \( A(n', n), c', p, \) and \( t \) designate, respectively, the \( n' \rightarrow n \) radiative transition probability and the rates of ionization, photo- and three-particle recombination, while the subscripts \( \text{nuc} \) and \( \text{H} \) refer, respectively, to the nucleus and the [\( \text{H} \) ] ion. Considering that the relaxation times of ES populations are, as a rule, much smaller than those of the ground-state populations, Bates, Kingston, and McWhirter [1] and McWhirter and Hearn [2] proposed that the derivatives of only these latter should be retained, while the ES populations are to be determined through the solution of a set of linear algebraic equations (the balance equations) taking the populations of the ground states to be given:

\[
\frac{dN_z(n)}{dt} = S_z(n) = 0 \quad (n \geq 2).
\]

Therein lies the quasisteady approximation for ES population densities. In this case the ES populations are determined entirely by the instantaneous values of the plasma parameters: the electron temperature, the density, and the ion composition. The necessary (but not sufficient) condition for the quasisteady approximation to be appropriate for ES populations is

\[
\sum_{n \geq 2} N_z(n) << N_z(1), N_{z+1}(1), N_e.
\] (1)

The gradient-scale time for the plasma parameters \( \tau = N_e / |dN_e/dt| \) (or \( T_e / |dT_e/dt| \)) should be many times higher than the period to establish the quasisteady population densities. Since the total decay rate \( W_n = A(n) + N_e \left( \sum_{n' \neq n} c(n, n') + c'(n) \right) \) of level \( n \) can serve as a measure of the latter relaxation process, the condition \( \tau >> W_n^{-1} \) must hold. With hydrodynamic flow, the relevant gradient-scale length may appear in this condition since