A COMPARISON OF MEASURED AND CALCULATED ELASTIC ANISOTROPIES OF MARBLE

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1. INTRODUCTION

The classical methods for calculating the elastic parameters of monomineral aggregates as quasi-isotropic material were published by Voigt [1] and Reuss [2] for cases without any preferred orientation. Voigt [1] determined the elastic constants of an aggregate by averaging the elastic constants of all grains. On the other hand, Reuss [2] proposed the same procedure for elastic moduli. Both methods are approximate. Voigt's procedure contains the assumption that the strain is uniform throughout an aggregate, whereas Reuss's that the stress is uniform. Hill [3] has shown that the values of the real moduli lie between the values given by both the other authors.

In both cases uniform orientation, i.e. random arrangement of crystallographical axes of grains in space, is considered. Under this assumption the aggregate is isotropic. But grains in real rocks have no accidental arrangement, in most cases a preferred orientation can be observed, which is one of the main causes of the anisotropy of rocks.

The endeavour has appeared recently, on the one hand, to improve the Voigt and the Reuss methods of calculations of the elastic parameters of an isotropic aggregate [4] and, on the other hand, to take into consideration the anisotropy caused by the structure of the rock. Calculations for the most simple models of preferred orientation were published [5].

The purpose of this paper is to calculate the elastic constants and the velocities of P-waves for the anisotropic monomineral aggregate represented by marble, and to compare the calculated velocities with the real velocities obtained by ultrasonic measurements on the sample. The calculation is based on the observed orientation of optical axes of calcite in the marble sample and on the elastic constants of calcite.

2. CALCULATION OF ELASTIC CONSTANTS

The experimental data, i.e. the diagram of preferred orientation and the measured velocities of P-waves, were given in [6, 7]. The velocities of longitudinal waves were measured on a spherical sample of marble in 133 independent directions. The orientation of calcite grains was determined by the directions of the optical axes measured microscopically on the universal stage.

Considering the fact that the orientation of crystallographic axes perpendicular to the optical axis was not determined, the position of the grains in space is not determined unambiguously. The assumption that the orientation of axes perpendicular to the optical axis is random was adopted. This assumption was satisfied by substituting the real calcite grain by a grain of a higher symmetry obtained by averaging the elastic parameters during the rotation of the calcite grain around its optical axis. Let us

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call such a grain “rotationally symmetric”. Its orientation in space is determined unambiguously by the orientation of its optical axis. The elastic parameters of marble were calculated for an aggregate composed of “rotationally symmetric” grains, the optical axes orientations of which were taken into consideration.

2.1 The Elastic Constants of the “Rotationally Symmetric” Grain

The elastic constants of a “rotationally symmetric” grain were obtained by averaging the elastic constants of calcite in the course of rotation around its optical axis through an angle of 360°.

Let us denote the optical axis of a calcite grain by x3. Then the components of the transformation tensor $g_{ij}$, which expresses the rotation around the optical axis through an angle $\varphi$ are

$$g_{ij} = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The elastic constants $c_{ijkl}$ form a tensor of rank four and they can be transformed as follows:

$$\tilde{c}_{ijkl} = g_{im}g_{jn}g_{kp}g_{lr}c_{mnpr}.$$

Let us denote the elastic constants of the “rotationally symmetric” grain by $c_{ijkl}$, then

$$c_{ijkl} = \frac{1}{2\pi} \int_0^{2\pi} \tilde{c}_{ijkl} \, d\varphi.$$

or, substituting from Eq. (2),

$$c_{ijkl} = (2\pi)^{-1} \int_0^{2\pi} g_{im}g_{jn}g_{kp}g_{lr} \, d\varphi.$$

From Eq. (4), after the substitution from Eq. (1) we obtain 5 independent elastic constants of the “rotationally symmetric” grain. If we use the matrix notation with two suffixes $i, j$ ($i = 1, 2, \ldots, 6; j = 1, 2, \ldots, 6$), then $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ are the independent constants mentioned above. For the dependent constants it follows that

$$c_{11} = c_{22}, \quad c_{13} = c_{23}, \quad c_{44} = c_{55}, \quad c_{66} = \frac{1}{2}(c_{11} - c_{12}).$$

The other constants are equal to zero. The “rotationally symmetric” grain has 9 non-zero constants and, according to Eq. (5), it is of hexagonal symmetry.