GLOBAL GEOID SECTIONS DETERMINED BY SATELLITE ORBIT DYNAMICS

Milan Burša

Research Institute of Geodesy, Prague*)

Summary: This paper is based on the fundamental notations of which have been adopted. The sets of Stokes' constants (harmonic coefficients) \( J_n^{(l)} \), \( S_n^{(k)} \) were adopted from [4], and the scale factor for lengths, \( R_0 = GM/W_0 \), from [5]. The equations for global meridional and parallel sections of the geoid surface \( W = W_0 \) are formulated. The geoid sections are represented by best fitting ellipses, as regards the meridians always for the arcs between the equator and the pole.

1. MERIDIONAL SECTIONS OF THE SURFACE \( W = W_0 \)

The difference between the geocentric radius-vectors \( q \) of the surface \( W = W_0 \) and \( q_c \) of a rotational ellipsoid \( a, e^2 \), the co-ordinates of the centre of which with respect to the geocentre are \( \Delta x_0, \Delta y_0, \) and \( \Delta z_0 \), is

\[
q - q_c = x_0^{(0)} - \Delta x_0 \cos \phi \cos \lambda - \Delta y_0 \cos \phi \sin \lambda - \Delta z_0 \sin \phi + \\
+ \sum_{n=2}^{\infty} \sum_{k=0}^{n} (a_n^{(k)} \cos k\lambda + \beta_n^{(k)} \sin k\lambda) P_n^{(k)}(\sin \phi).
\]

The expressions for the coefficients in Eq. (1) follow from [5, 7]; if the terms of the order of \( 10^{-8} \) are neglected, then

\[
\begin{align*}
\alpha_0^{(0)} &= R_0 \left[ 1 + \frac{1}{3} \left( \frac{a_0}{R_0} \right)^{-3} q + \frac{2}{5} \left( \frac{a_0}{R_0} \right)^{-6} q^2 - \frac{1}{15} \left( \frac{a_0}{R_0} \right)^{-1} J_2^{(0)} q - \frac{2}{5} \left( \frac{a_0}{R_0} \right)^{4} \left( J_2^{(0)} \right)^2 \right] - \\
&- a \left( 1 - \frac{1}{6} e^2 - \frac{11}{120} e^4 - \frac{103}{1680} e^6 \right), \\
\alpha_2^{(0)} &= R_0 \left[ \left( \frac{a_0}{R_0} \right)^2 \left( J_2^{(0)} \right)^2 - \frac{1}{3} \left( \frac{a_0}{R_0} \right)^{-3} q - \frac{4}{7} \left( \frac{a_0}{R_0} \right)^{-6} q^2 + \frac{5}{21} \left( \frac{a_0}{R_0} \right)^{-1} J_2^{(0)} q - \\
&- \frac{4}{7} \left( \frac{a_0}{R_0} \right)^{4} \left( J_2^{(0)} \right)^2 \right] - a \left( - \frac{1}{3} e^2 - \frac{5}{42} e^4 - \frac{3}{36} e^6 \right), \\
\alpha_4^{(0)} &= R_0 \left[ \left( \frac{a_0}{R_0} \right)^4 \left( J_4^{(0)} \right)^2 + \frac{6}{35} \left( \frac{a_0}{R_0} \right)^{-6} q^2 - \frac{6}{35} \left( \frac{a_0}{R_0} \right)^{-1} J_2^{(0)} q - \frac{36}{35} \left( \frac{a_0}{R_0} \right)^{4} \left( J_2^{(0)} \right)^2 \right] - \\
&- a \left( \frac{3}{35} e^4 + \frac{57}{770} e^6 \right),
\end{align*}
\]

*) Address: Politických vězňů 12, Praha 1 - Nové Město.
Global Geoid Sections Determined by Satellite Orbit Dynamics

\[ \alpha_n^{(0)} = A_0^{(0)} = R_0 \left( \frac{a_0}{R_0} \right)^n J_n^{(0)}, \quad n = 3, 5, 6, \ldots, \]

\[ \beta_n^{(k)} = A_n^{(k)} = R_0 \left( \frac{a_0}{R_0} \right)^n J_n^{(k)} S_n^{(k)}, \quad n = 2, 3, \ldots; \quad k = 1, 2, \ldots, n. \]

From Eq. (1) it is easy to get the components \( \xi, \eta \) of the deviations of the normals to the surfaces considered (\( B \) – latitude with respect to the ellipsoid \( a, e^2 \) in question)

\[ \begin{align*}
\xi'' &= - \frac{q''}{M} \frac{\partial (q - q_e)}{\partial \Phi} \frac{\partial \Phi}{\partial B}, \\
\eta'' &= - \frac{q''}{N \cos B} \frac{\partial (q - q_e)}{\partial \Lambda}, \\
M &= a(1 - e^2)(1 - e^2 \sin^2 B)^{-3/2}, \\
N &= a(1 - e^2 \sin^2 B)^{-1/2}, \\
\frac{d\Phi}{dB} &= 1 - e^2 + 2e^2 \sin^2 \Phi + e^4 \sin^4 \Phi.
\end{align*} \]

If we also neglect expressions of the order of \( e^2 a^{-1} \Delta x_0, e^2 a^{-1} \Delta y_0, e^2 a^{-1} \Delta z_0, e^4 J_2^{(0)}, e^4 q, e^4 J_2^{(k)} (n > 2), e^4 S_n^{(k)}, \) etc., with a view to Eq. (1), and to the following,

\[ \begin{align*}
\sin^2 B &= \sin^2 \Phi \left[ 1 + 2e^2 \cos^2 \Phi(1 - 2 \sin^2 \Phi) + 3e^4 \cos^2 \Phi \right], \\
\cos^2 B &= \cos^2 \Phi \left[ 1 - 2e^2 \sin^2 \Phi(1 + 2 \cos^2 \Phi) + e^4 \sin^2 \Phi \right], \\
aM^{-1} &= 1 + e^2 \left( 1 - \frac{5}{2} \sin^2 \Phi \right) + e^4 \left( 1 - \frac{9}{4} \sin^2 \Phi + \frac{27}{8} \sin^4 \Phi \right), \\
aN^{-1} &= 1 - \frac{1}{2}e^2 \sin^2 \Phi - e^4 \sin^2 \Phi(1 - \frac{7}{8} \sin^2 \Phi).
\end{align*} \]

Eq. (2) can be expressed as

\[ \begin{align*}
\xi'' &= - (q''/a) \sum_{n=2}^{\infty} \sum_{k=0}^{n} (\alpha_n^{(k)} \cos k\lambda + \beta_n^{(k)} \sin k\lambda) \left( 1 + \frac{1}{2}e^2 \sin^2 \Phi + \frac{3}{8}e^4 \sin^4 \Phi \right), \\
\eta'' &= (q''/a) \sum_{n=2}^{\infty} \sum_{k=1}^{n} k(\alpha_n^{(k)} \cos k\lambda - \beta_n^{(k)} \sin k\lambda) \sec \Phi \Phi_n^{(k)}(\sin \Phi); \\
n\Phi_n^{(k)}(\sin \Phi) \end{align*} \]

the expression for \( d\Phi_n^{(k)}(\sin \Phi)/d\Phi \) is given in [6]. The heights \( \xi \) and the components \( \xi, \eta \), computed from the set of harmonics \( J_n^{(k)}, S_n^{(k)} \) [4], considering \( R_0 = 6363675 \) m [5], with respect to the IAG 1967 ellipsoid (\( a = 6378160 \) m, \( e^2 = 0.00669460 \), \( \Delta x_0 = 0, \Delta y_0 = 0, \Delta z_0 = 0 \), are represented in Figs. 1–4.

Taking \( A = \text{const.} = A_{\infty} \), with a view to [5], we arrive at the following expression for the radius-vector of the meridional section

\[ \alpha_r = A_0^{(0)} + \sum_{n=2}^{\infty} \sum_{k=0}^{n} (a_n^{(k)} \cos \lambda \lambda + b_n^{(k)} \sin \lambda \lambda) \Phi_n^{(k)}(\sin \Phi). \]

We wish to determine the parameters \( a_r, e_r \) of the best fitting ellipse, as well as the