ON A NEW CONDITION FOR DIRECTIONS
OF A SPATIAL NETWORK

JOSEF KABELĂČ

Astronomical and Geophysical Observatory, The Czech Technical University, Prague*)

Zusammenfassung: Es wird eine neue Bedingung, Gl. (6), angeführt, die von den bereits aus-
geglichenen, z. B. durch den Stundenwinkel t und die Deklination δ definierter Richtungen der
Seiten des Raumnetzes erfüllt werden muss. Zur Demonstration dieser Bedingung wird ein Modell des
Raumnetzes in der Form eines Tetraeders benutzt (Abb. 1).

This paper belongs to the branch which is concerned with adjusting satellite spatial networks
derived from Earth satellite observations, i.e. to satellite geodesy. By means of photographic
simultaneous, or quasi-simultaneous, observations from points A and B on the surface of the
Earth, the topocentric equatorial co-ordinates of a satellite, or of any other artificial body, are
obtained, and from these the equatorial co-ordinates of the line joining A and B [1], or the direc-
tional cosines of this line in the system of astronomical co-ordinates. These directions are either
obtained by adjusting the intermediate observations, exploiting the condition of coplanarity of three
directions (see, e.g., [2]), or by adjusting according to intermediate observations, making use
of rectangular, spatial, referential, geodetic co-ordinates of satellite points of the network (see,
e.g., [3, 4]). Also the methods of introducing weights differ mutually.

In order to obtain the directions of the joins of satellite points, it is usual to apply common
adjustment of the joints of the points of the satellite, spatial network. This problem is then divided
into two stages [2–4], although in [5] a theoretical possibility of simultaneous adjustment of both

*) Address: Karlovo nám. 13, Praha 2.

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observations can be used. If the adjustment is conducted according to [3, 4] et al., two systems
are in fact being joined: the geodetic and the astronomical. From the geodetic system the co-
ordinates of the satellite points are adopted, to which corrections are added, from the astronomical
system the topocentric equatorial co-ordinates of the joins of the satellite network points, although
these two systems differ in orientation [6, 7]. The previous directional conditions are joined by
metric conditions. These can follow from the distances determined between points of the satellite
orbit [8], between the satellite and the satellite points of the network [9], or between the satellite
points, derived from geodetic measurements on the Earth’s surface.

The paper only considers the relations between the directional characteristics of the joints of
the satellite spacial network points. The condition of complanarity between 3 directions has been
expanded theoretical to an arbitrary number of directions of a given network. The newly derived
relation must satisfy the directions already adjusted. In order to demonstrate this condition
a model of the spacial network in the shape of a tetrahedron has been used. In Fig. 1, \( \mathbf{p}_i \) represent
vectors of the i-th side of the network, for \( i = 1, 2, ..., 6 \). Let us write two conditions, e.g.,

\[
\begin{align*}
\mathbf{p}_1 - \mathbf{p}_3 + \mathbf{p}_5 - \mathbf{p}_6 &= 0, \\
\mathbf{p}_2 + \mathbf{p}_4 - \mathbf{p}_5 - \mathbf{p}_6 &= 0,
\end{align*}
\]

which define the shape of the tetrahedron PRNU. Let us introduce \( \mathbf{p}_i' = \beta_i \mathbf{p}_i \) for \( i = 1, 2, ..., 6 \),
where \( \mathbf{p}_i' \) has a direction identical with vector \( \mathbf{p}_i \), but a different magnitude and \( \beta_i \) is a scalar
different from zero, into Eqs. (1). Both can be divided by scalar \( \beta_6 \) allowing for new unknowns to be
introduced: \( x_j = \beta_j / \beta_6 \) for \( j = 1, 2, ..., 5 \). Equations (1) then become

\[
\begin{align*}
x_1 \mathbf{p}_1' + x_3 \mathbf{p}_3' + x_5 \mathbf{p}_5' - \mathbf{p}_6 &= 0, \\
x_2 \mathbf{p}_2' + x_4 \mathbf{p}_4' + x_6 \mathbf{p}_6' &= 0.
\end{align*}
\]

Equations (2) express the linear dependence of vectors \( \mathbf{p}_i' \), for which it holds that \( \mathbf{p}_i' = x_i \mathbf{i} +
+ y_i \mathbf{j} + z_i \mathbf{k} \), where \( x_i, y_i \) and \( z_i \) are scalar components of vector \( \mathbf{p}_i \) and \( \mathbf{i}, \mathbf{j} \) and \( \mathbf{k} \) are unit
vectors. Equations (2) can be resolved into components as follows:

\[
\begin{align*}
x_1 x_1 + x_3 x_3 + x_5 x_5 - x_6 &= 0, \\
x_2 y_2 + x_4 y_4 - x_5 y_5 - y_6 &= 0, \\
x_1 z_1 + x_3 z_3 + x_5 z_5 - z_6 &= 0, \\
x_2 z_2 + x_4 z_4 - x_5 z_5 - z_6 &= 0.
\end{align*}
\]

The system of Eqs. (3) contains 5 unknowns \( x_j \). By solving any 5 equations of this system it is
possible to express \( x_j \) and to substitute into the sixth equations. After modifications we obtain
a determinant of the 6th degree:

\[
\begin{vmatrix}
x_1 & x_3 & x_5 & x_6 \\
x_2 & x_4 - x_3 & x_6 \\
x_1 & y_3 & y_4 & y_6 \\
x_2 & y_4 & y_5 & y_6 \\
x_1 & z_3 & z_4 & z_6 \\
x_2 & z_4 & z_5 & z_6 \\
\end{vmatrix} = 0.
\]

This determinant is then also valid for scalars \( x_i, y_i, z_i \), which are not components of vectors \( \mathbf{p}_i \),
where \( |\mathbf{p}_i| \) is equal to the length of the i-th side of the tetrahedron PRNU. The scalars \( x_i, y_i, z_i \)

*) Eqs. (1) can also be reached in another way. Let us consider vectors \( \mathbf{p}_1, \mathbf{p}_2 \) and \( \mathbf{p}_6 \), Fig. 1,
to be independent. The remaining vectors \( \mathbf{p}_3, \mathbf{p}_4 \) and \( \mathbf{p}_5 \) are functions of the former: \( \mathbf{p}_3 =
= -\mathbf{p}_2 + \mathbf{p}_6, \mathbf{p}_4 = -\mathbf{p}_1 + \mathbf{p}_6, \mathbf{p}_5 = -\mathbf{p}_1 + \mathbf{p}_2 \). By adding the first to the third we obtain
\( \mathbf{p}_1 + \mathbf{p}_3 + \mathbf{p}_5 - \mathbf{p}_6 = 0 \) and by subtracting the third from the second \( \mathbf{p}_2 + \mathbf{p}_4 - \mathbf{p}_5 - \mathbf{p}_6 = 0 \)
which represent Eqs. (1).