A CONTRIBUTION TO THE STUDY OF THE BAROTROPIC
MODEL OF THE ATMOSPHERE

PART 2

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CHANGES IN HEIGHT OF LEVEL OF NON-DIVERGENCE

Let us assume that in the troposphere there exists only one level of non-divergence which coincides with the isobaric surface \( p = p_2 \), and let us further assume that the isobaric divergence is a linear function of the pressure. The isobaric divergence is then expressed by the relation

\[
\nabla_p \cdot v = \nabla_p \cdot v_0 \frac{p - p_2}{p_0 - p_2}.
\]

(27)

In Eq. (27) \( v_0 \) denotes the wind vector in the isobaric surface \( p_0 = 1000 \text{ mb} \). The assumptions used express the simple but typical example which, in some circumstances, is suitable for the description and study of real synoptic processes.

The tendency equation of absolute topography 1000 mb can be written in the form

\[
\frac{\partial \Phi_0}{\partial t} = -v_0 \cdot \nabla_p \Phi_0 - \frac{RT_0}{p_0 - p_2} \int_0^{p_2} \nabla_p \cdot v \, dp.
\]

(28)

If we substitute for \( \nabla_p \cdot v \) from equation (27) we obtain

\[
\frac{\partial \Phi_0}{\partial t} + v_0 \cdot \nabla_p \Phi_0 = -\frac{RT_0}{p_0 - p_2} \left( \frac{1}{2} p_0 - p_2 \right) \nabla_p \cdot v_0.
\]

Let us now consider the difference between the divergence in the levels \( p_0 \) and \( p_1 (p_0 > p_1) \). We thus obtain the relative divergence \( \nabla_p \cdot v' \), which was introduced to meteorology by Sutcliffe [5]. If we choose the difference \( p_0 - p_1 \) so that it represents a sufficiently thick and representative layer in the troposphere, then the relative divergence denotes the development in this layer, or in the whole troposphere. By means of Eq. (27) we easily find that it holds that

\[
\nabla_p \cdot v = \frac{p_0 - p_2}{p_1 - p_2} \nabla_p \cdot v',
\]

so that

\[
\frac{\partial \Phi_0}{\partial t} + v_0 \cdot \nabla_p \Phi_0 = \frac{RT_0}{p_0 - p_1} \left( \frac{1}{2} p_0 - p_2 \right) \nabla_p \cdot v'.
\]

From this it follows that

\[
p_2 = \frac{1}{2} p_0 - \frac{RT_0 \nabla_p \cdot v'}{\frac{RT_0}{p_0 - p_1}} (p_0 - p_1).
\]

If we further assume that the following inequality is fulfilled

\[
\left| \frac{\partial \Phi_0}{\partial t} \right| \gg \left| v_0 \cdot \nabla_p \Phi_0 \right|,
\]

(29)

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we obtain the resultant relation for the position of the level of non-divergence in the form

$$p_2 = -\frac{1}{2} p_0 \left(G_0 - G_1\right) \frac{\partial \Phi_0}{\partial t} R T_0 \nabla_p \cdot v'$$  \hspace{1cm} (30)$$

Equation (30) determines the position of the level of non-divergence as a function of the relative divergence and the surface-pressure changes. The quantities $\frac{\partial \Phi_0}{\partial t}$ and $\nabla_p \cdot v'$ are not of course generally independent and it should be possible to derive relations which mutually connect them. This procedure would however not have any particular meaning for our purposes.

If the inequality (29) is not fulfilled, we can write

$$\frac{\partial \Phi_0}{\partial t} + v_0 \cdot \nabla_p \Phi_0 = \left(\frac{\partial \Phi_0}{\partial t}\right)_d,$$

where the symbol $\left(\frac{\partial \Phi_0}{\partial t}\right)_d$ denotes the dynamic surface-pressure changes in the sense of the terminology proposed by Brandejs, Kopáček, Vitek and Zikmunda [6]. In Eq. (30) we then work with dynamic changes in the absolute topography 1000 mb instead of actual changes. For the sake of simplicity we shall speak below of surface-pressure changes without more detailed specification and in this notation will be included either real changes or dynamic changes according to whether inequality (29) is satisfied or not.

Let us assume in the first approximation that $\frac{\partial \Phi_0}{\partial t} = 0$. Then $p_2 = -\frac{1}{2} p_0$.

Thus, on the assumption that the surface-pressure changes are zero, the level of non-divergence is identical with the isobaric surface 500 mb. All the results following from an analysis of Eq. (30) of course are based on the assumption of linear dependence of the divergence on pressure and must be taken with a certain reserve, since the real relations in the atmosphere can be far more complicated. On the other hand, however, we must allow that the discussions of this equation can at least provide valuable orientational data on the mechanism of changes in height of the level of non-divergence.

It is obvious from Eq. (30) that the level of non-divergence lies below the level 500 mb if the surface-pressure changes have opposite sign to the relative divergence. The fraction on the right side of Eq. (30) is then negative so that $p_2 > -\frac{1}{2} p_0$. This corresponds to situations with a typically development character (e.g. positive relative divergence denoting cyclogenesis is connected with decreases in surface pressure).

The condition for the coincidence of the level of non-divergence with the isobaric surface 1000 mb is

$$\frac{\partial \Phi_0}{\partial t} = -\frac{R T_0 p_0}{2 (p_0 - p_1)} \nabla_p \cdot v'.$$

By estimating the order of magnitude we easily see that the level of non-divergence can be identical with the level 1000 mb only for fairly small values of the relative divergence. At the same time it is seen that the level of non-divergence can decrease to sea-level only if the surface-pressure changes have different sign to the relative divergence. These two conditions can be simultaneously satisfied probably in the highest development stage of pressure systems, when for example a cyclone still somewhat deepens