A RAPID ASTRONOMICAL METHOD OF OBSERVATION FOR GEODETIC AZIMUTH AND PRIME VERTICAL DEFLECTION OF THE VERTICAL

JNANENDRA CHANDRA BHATTACHARJI
Survey of India, Dehra Dun


1. INTRODUCTION

The deviation of the vertical having the meridional deflection \( \zeta \) as \( (\phi_a - \phi_g) \), and the prime vertical deflection \( \eta \) as \( (\lambda_a - \lambda_g) \cos \varphi \), where \( \varphi \) and \( \lambda \) denote latitude and longitude and the suffixes \( a \) and \( g \) stand for astronomical and geodetic, has the following components: \( \eta \sin A + \zeta \cos A \) and \( \eta \cos A - \zeta \sin A \) along and perpendicular to the direction of azimuth \( A \). As a result the directions indicated by an observing theodolite in the plane containing the normal to the geoid, require corrections similar to those needed for the tilt of the horizontal axis of the theodolite, for reducing them to the plane containing the normal to the spheroid of reference.

Hence the astronomical azimuth of the celestial pole needs a correction of \( -\eta \tan \varphi \), which is constant for the latitude of the observation station, the component of the deviation of the vertical in the direction perpendicular to that of the pole being \( \eta \) and the altitude of the pole \( \varphi \). In fact this correction represents the angle between the geodetic and the astronomical north lines and is, therefore, to be applied like some sort of index correction to all astronomical azimuths in addition to that ordinarily required for the altitude of the point observed, in order to reduce them to geodetic azimuths. The above correction is known as the Laplace correction, and stations at which such Laplace corrections become known are called Laplace stations. Since, in an extensive survey, the computation of geodetic azimuths involves a greater accumulation of error than that of latitudes or longitudes, it is essential to establish Laplace stations at frequent intervals in order to check the geodetic azimuths of the survey in the same way as bases check its scale [1].

New close to the equator where \( \varphi \) is small, the Laplace correction becomes inappreciable and as such observations of astronomical azimuths can be used to check the survey directly. But in moderate latitudes, the effect of the deflections of the vertical becomes significant, and "in high latitudes it would appear at first sight that it is impracticable to apply the Laplace correction since it is not possible to determine longitude accurately, and the azimuths are affected with nearly the full uncertainty in the longitude" [4]. The difficulties can, however, be easily avoided as origi-
nally suggested by Black [4], if, instead of astronomical azimuths of stars, we observe directly for
geoedetic azimuths by solving the spherical triangle $SPZ$ in the adjoining diagram, using $90^\circ - \phi_0$
for $PZ$ and the hour angle as deduced from the geodetic longitude for $t$, and then reduce the
corresponding geodetic azimuths of the referring marks at nearly zero elevation from the stations
of observation, by applying a correction of $\tan h(\eta \cos A - \zeta \sin A)$ to the observed horizontal
angles between the stars of observation and the referring marks, where $h$ is the altitude and $A$ the
azimuth of stars of observation. The correction being thus dependent on the altitude and the
azimuth of the stars of observation, it can be reduced to a negligible quantity by means of a proper
selection of the stars of observation. This is exactly what has actually been done in the method
described in the present article by considering observations of only north-south pairs of stars at
appropriate azimuth circle readings near the observer’s meridian.

2. DESCRIPTION OF THE METHOD

Let us consider a point $R$ of altitude $h$. Then the correction needed to reduce its
astronomical azimuth to the geodetic azimuth, will be $- \eta \tan \varphi + \tan h(\eta \cos A -
-\zeta \sin A)$, where $A$ denotes the azimuth of the point and $\eta \cos A - \zeta \sin A$
stands for the component of the deviation of the vertical in the direction perpendicular to the line of pointing. Similarly, if we consider another point $S$ of
altitude $h'$, and in the direction of azimuth $A'$, the corresponding correction to reduce
the astronomical azimuth to the geodetic azimuth, will be $- \eta \tan \varphi +
+ \tan h'(\eta \cos A' - \zeta \sin A')$. Hence the resultant correction needed to reduce the
observed horizontal angle $\Theta$, say, between $R$ and $S$ to the corresponding geodetic
angle, becomes $\tan h'(\eta \cos A' - \zeta \sin A') - \tan h(\eta \cos A - \zeta \sin A)$. Obviously,
on adding this reduced geodetic angle to the geodetic azi-
muth of $S$, we obtain straight away the geodetic azimuth
of $R$. Now if $S$ be a heavenly body of altitude $h'$, and $R$
a reference mark at a negligibly small elevation from the
station of observation, the geodetic angle between $R$ and $S$ becomes $\Theta + \tan h'(\eta \cos A' - \zeta \sin A')$, which
is a function of the altitude and azimuth of the heavenly
body $S$. As is evident from the spherical triangle $SPZ$
(Fig. 1), we can compute the geodetic azimuth of $S$ by the
observing time and horizontal angles, and using geo-
detic latitude for the side $ZP$ and hour angle $t$ as deduced
from the geodetic longitude. If we add the geodetic angle
$\Theta + \tan h'(\eta \cos A' - \zeta \sin A')$ to this computed geodetic azimuth of $S$, we obtain
as before the geodetic azimuth of the reference mark $R$.

Let $S_N$ and $S_S$ be a pair of north and south stars of altitudes $h_N$ and
$h_S$ (not differing considerably from each other) and lying close to the meridian so that their azimuths
are $-A_1$, $0^\circ$, $A_2$, $180^\circ - A_3$, $180^\circ$ and $180^\circ + A_4$, where $A_1, \ldots, A_4$ are small and
nearly equal to one another. If the observed horizontal angles between the reference