AMPLITUDE OF A REFLECTED WAVE IN A HORIZONTALLY STRATIFIED MEDIUM

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1. INTRODUCTION

Papers [1, 2] treated the computation of the travel-time curve and of the amplitude curve of a refracted wave in a medium with a general velocity-depth pattern by substituting this medium by a horizontally stratified medium with simple velocity patterns in the individual layers. In the paper presented, attention is devoted to the computation of the amplitude curve of a reflected wave from the same point of view. The amplitude in question is that of the vertical component of displacement, computed on the basis of the zero approximation of the ray theory. The substituting medium is considered to be a stratified medium with a constant velocity in the layers, on the one hand, and a stratified medium with a linear velocity variation in the layers, on the other.

2. THEORETICAL PART

The parameters of the medium, the co-ordinate system, the source and point of recording are considered to be the same as in [1]. The parameter of the ray of the reflected wave is again the angle \( \psi \), between the ray and the vertical at the exit from the source. The distance and the amplitude of the reflected wave will be computed as the function of parameter \( u (u = \sin \psi) \).

The total epicentral distance \( r \), at which the ray of the wave is incident at the surface of the medium, can be determined as the sum of the contributions \( \Delta r \) of the individual layers, through which the ray propagates downwards, multiplied by two due to the symmetry of the descending and ascending part of the path. If the ray is propagating through the \( i \)-th layer, which has a constant velocity, we obtain for the horizontal projection \( \Delta r_i \) the expression

\[
\Delta r_i = H_i a_i u \sqrt{(a_i^2 - a_i^2 u^2)},
\]

where \( H_i \) is the thickness of the \( i \)-th layer, \( a_i \) the velocity in the \( i \)-th layer and \( a_1 \) the velocity in the first layer. If the layer displays a linear velocity variation, we obtain

\[
\Delta r_i = H_i (a_{2i} + a_{2i-1}) u \left[ \sqrt{(a_i^2 - a_{2i}^2 u^2)} + \sqrt{(a_i^2 - a_{2i-1}^2 u^2)} \right],
\]

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where $a_{2i-1}$ is the velocity immediately under the upper boundary, $a_{2i}$ the velocity immediately above the lower boundary of the $i$-th layer, and $a_i$ the velocity immediately under the upper boundary of the first layer.

For the vertical component $V$ of the displacement of a reflected wave, applying the zero approximation of the ray theory (for the source as well as for the recording point on the surface of the medium) we obtain \[4\]

$$V = \delta \gamma \prod_{k=1}^{j} \lambda_k \sqrt{\nu^{-1}(1 - u^2)} (\partial r/\partial \psi_1),$$

where $\delta$ is the conversion coefficient, $\gamma$ is the reflection coefficient of a plane wave, $\lambda_k$ is the product of the refractive coefficients of a plane wave, i.e. the refractive coefficient going down and the refractive coefficient going up for the $k$-th interface. All the interfaces, through which the ray propagates before reaching the reflecting interface, must be included in the product. The individual quantities in Eq. (3) are given in [1, 3].

The expression $\partial r/\partial \psi_1$ can be obtained as the sum of contributions $\partial r/\partial \psi_1$ from all layers, through which the ray propagates on its way downwards, multiplied by two, again with a view to the symmetry of the path. By differentiating (1), we arrive at

$$\partial \Delta r_i/\partial \psi_1 = a_i^2a_iH_i(1 - u^2)^{1/2}/\sqrt{(a_i^2 - a_i^2u^2)};$$

by differentiating Eq. (2)

$$\frac{\partial \Delta r_i}{\partial \psi_1} = \frac{H_i a_i^2 (a_{2i} + a_{2i-1}) (1 - u^2)^{1/2}}{\sqrt{(a_i^2 - a_{2i-1}^2u^2)} \sqrt{(a_i^2 - a_{2i}^2 u^2)}}.$$

As all the rays of the reflected wave (with the exception of the last ray, which reaches maximum epicentral distance), are at an angle to the vertical at all layer boundaries which is smaller than $90^\circ$, all the contributions $\partial \Delta r_i/\partial \psi_1$ are positive and finite. In contrast to the refracted wave [1], therefore, no discontinuities or singularities, resulting from the discontinuity of the velocity gradients at the layer boundaries, occur on the amplitude curve of the reflected wave. For a reflected wave, therefore, it is not necessary to apply a stratified medium with continuous velocity gradients at the layer boundaries.

3. EXAMPLE

In order to illustrate the possibilities of substituting the given medium by a stratified medium, the amplitude curves for one example have been computed. The given medium is a model, 10 km thick, in which the velocity varies with depth as $a(z) = 0.05263(1 + 152z)^{1/2} + 3.94737$. The velocity at the top is, therefore, equal to 4 km/s and at the bottom to 6 km/s. The model is located on a half-space with a constant velocity of either 6.2 km/s or 7 km/s. The boundary dividing the model from the half-space is the boundary at which the wave considered is reflected. The density is constant and $a(P)/a(S) = \sqrt{3}$. 