TRANSFORMATION OF PLATE CO-ORDINATES TO EQUATORIAL CO-ORDINATES

Dedicated to Professor František Fiala on His 85th Birthday

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1. INTRODUCTION

In 1963 experiments were begun at the Research Institute of Geodesy, Topography and Cartography with determining the positions of artificial Earth satellites from photographs. Owing to the fact that no special instruments were available for this purpose in Czechoslovakia, an older type of aerial camera Rb with a focal length of 750 mm was adapted. It is a film camera of the 30 by 30 cm size. Each observation post was equipped with an apparatus comprising two of these cameras. The cameras are fitted with louvre shutters, the opening of which is controlled by the electronic unit of the apparatus. This unit is set apart from the cameras, communications being maintained by electric cables. The author of this paper, besides presenting an analysis of the accuracy of the apparatus, from the point of view of photogrammetry [2], has attempted to find a convenient method of transformation of plate co-ordinates to equatorial co-ordinates.

2. SOME METHODS OF PLATE CO-ORDINATE TRANSFORMATION

First of all Turner's method should be mentioned. This method is well known in astrometry and has been described in various text books, e.g. [4]. A different method of calculation was elaborated in Czechoslovakia by Ceplecha, and it is being used for determining the positions of meteors [1]. Ceplecha has lately developed this method to use in the determining of satellite positions. The case where neither the exterior orientation, nor the interior orientation of the photographic camera is known, and both have to be determined by simultaneous calculations, is called total orientation by Merritt [3].

The transformations mentioned are all carried out in a plane. Turner's method, for example, looks for a relation between image co-ordinates and ideal image co-ordinates, which are obtained by projecting stars onto the ideal image plane by means of the image principal point. If the calculation of the required values is being carried out in this method by adjustment, small positional deviations in the image plane are sought. The aim of the calculations is not to determine the position of points in a plane, nor in space, but to determine directions only. It is, therefore, useful to apply a method of calculation and adjustment, which will result in minimum angular deviations of the directions.

3. SPACIAL AFFINE TRANSFORMATION

3.1 Deformation of the pencil of light

A pencil of light from objects on the celestial sphere, entering the lens of a photographic camera, is deformed during the photographic process. Owing to difference in shrinkage of the film strip in the production direction of the film and perpendicular to it, the pencil of light on

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transformation from the photograph, becomes affinely deformed. The affine deformation of a pencil of light on reconstruction is also the result of errors in the camera focal length. These systematic deformations of a pencil of light can be embodied in calculations of the pencil transformation, and it is, therefore, useful to adopt affine transformation for this purpose. A systematic pencil of light deformation, caused by lens distortion, is best compensated in advance by correcting the plate co-ordinates. From now on, plate co-ordinates $x, y$ are to be considered as corrected for lens distortion. Further deformations of the pencil of light can be caused, e.g. by irregular local deformations of the film strip, irregularities in the lens distortion, or other influences. These random deformations, when a surplus of observations is available, can be the subject of adjustment.

3.2. Transformation of plate co-ordinates

For the transformation of plate co-ordinates to equatorial co-ordinates, after certain modifications which will be explained later on, equations for the affine spatial transformation of rectangular co-ordinates can be used

$$
\begin{align*}
\bar{X} &= a_1 x + b_1 y + c_1 z, \\
\bar{Y} &= a_2 x + b_2 y + c_2 z, \\
\bar{Z} &= a_3 x + b_3 y + c_3 z.
\end{align*}
$$

Let us demonstrate the way in which the relations given can be used for the transformation of plate co-ordinates to equatorial co-ordinates. The origin of both co-ordinate systems, $X, Y, Z$ and $x, y, z,$ will be in the centre of the camera lens, i.e. in the centre of the entrance pupil. The first system will have the $Z$-axis pointing to the celestial pole, the $X$-axis towards the vernal node and the $Y$-axis perpendicular to the $XZ$-plane. It is a right-handed system (Fig. 1). The following relation holds between the spherical equatorial co-ordinates $x, \delta$ and the spatial coordinates $X, Y, Z$ on a unit sphere

$$
\begin{align*}
X &= \cos \delta \cos x, \\
Y &= \cos \delta \sin x, \\
Z &= \sin \delta.
\end{align*}
$$

As the points on the satellite trace and on the star traces cannot be exposed at the same time, it is more suitable to use hour angles than right ascension. The $X$-axis will then point to the intersection of the equator with the local meridian, and the angle of right ascension in Eq. (2) will be replaced by the hour angle $t$. In the second co-ordinate system, the $z$-axis is identical with the camera axis, and its positive part points to the projection of the principal point on to the celestial sphere. The directions of the $x$- and $y$-axes will be determined in each case by the line connecting two opposite fiducial marks, and their sense will be chosen in such a way as to render the system right-handed.

The quantities necessary for the transformation in Eq. (1) can be determined as follows.