ON THE STABILITY OF SOLUTIONS OF A QUASILINEAR UNCERTAIN SYSTEM

Yu. A. Martynyuk-Chernienko

UDC 531.36

We generalize the Lyapunov direct method, which can be used for establishing new conditions of the uniform asymptotic stability of solutions of an uncertain system with respect to an invariant moving set.

Introduction

In the present paper, we investigate models of uncertain systems that are described by quasilinear systems in a precisely specified linear approximation. Functions containing uncertain values of parameters realize the relation between certain linear subsystems.

One of the sources of "uncertainties" in the problem under consideration is the procedure of decomposition of a (real or formal) large-scale system with subsequent linearization. These uncertainties can be both internal and external. Internal uncertainties are related to the nature of each of the subsystems, while external uncertainties are a consequence of the interaction between subsystems.

Unlike the previous studies (see [1] and the references therein) of uncertain systems, this investigation is concerned with the dynamics of a quasilinear system with respect to a relatively moving invariant set. Here, we continue the investigations started in [2–4].

1. Statement of the Problem

Consider the motion of the uncertain system described by the quasilinear equations

\[ \frac{dz}{dt} = Pz + Q(z, w, \alpha), \]

\[ \frac{dw}{dt} = Hw + G(z, w, \alpha). \]

Here, \( z \in \mathbb{R}^n, w \in \mathbb{R}^m, \alpha \in \mathbb{R}^d \) is the uncertainty parameter of system (1), \( P, H \) are constant matrices of the corresponding dimensions, whose characteristic numbers \( \lambda_1, \ldots, \lambda_n \) and \( \beta_1, \ldots, \beta_m \) have prime elementary divisors, the components of \( Q : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^d \rightarrow \mathbb{R}^n \) and \( G : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^d \rightarrow \mathbb{R}^m \) are series in integer positive powers of \( z \) and \( w \). These series start from terms of order at least two and are absolutely convergent in the product of arbitrarily large open connected neighborhoods \( \mathcal{N}_z \) and \( \mathcal{N}_w \) of the states \( z = 0 \) and \( w = 0 \) for any values of the uncertainty parameter \( \alpha \in \mathbb{R}^d \).

We assume that the zero solution of the equations of the first approximation of system (1) is unstable and system (1) has a nonzero periodic solution for \( \alpha = 0 \). The aim of this paper is the formulation of sufficient conditions for uniform asymptotic stability of the solutions \( (z(t, \alpha), w(t, \alpha)) \) of system (1) with respect to the moving invariant set \( A^*(\alpha) \) to be defined below.

2. Transformation of the Uncertain System

By using the linear nonsingular transformation \( z = Tx \) and \( w = Ry \) (\( \det T \neq 0, \det R \neq 0 \)), we transform the linear part of system (1) to the diagonal form
ON THE STABILITY OF SOLUTIONS OF A QUASILINEAR UNCERTAIN SYSTEM

\[
\begin{align*}
\frac{dx}{dt} &= Ax + f_1(x, y, \alpha), \\
\frac{dy}{dt} &= By + f_2(x, y, \alpha),
\end{align*}
\]

(2)

where \( x \in \mathbb{R}^n, y \in \mathbb{R}^m, A = \text{diag}(\lambda_1, \ldots, \lambda_n), B = \text{diag}(\beta_1, \ldots, \beta_m), f_1(x, y, \alpha) = T^{-1}Q(Tx, Ry, \alpha), \) and \( f_2(x, y, \alpha) = R^{-1}G(Tx, Ry, \alpha). \)

In the system of equations (2), the components \( x_s, s = 1, 2, \ldots, n, \) corresponding to real eigenvalues \( \lambda_s \) will be real and those corresponding to complex eigenvalues will be complex. A similar statement is true for the variables \( y_k, k = 1, 2, \ldots, m. \) Every complex-conjugate pair of roots will correspond to a pair of complex-conjugate variables \( x_s, y_k. \) For real variables \( x_s \) and \( y_k \) we have \( x_s = \pm r_s, \theta_s = 0, \pi; \ y_k = \pm \rho_k, \varphi_k = 0, \pi. \)

For the components \( x_s \) and \( y_k \) of the vectors \( x \) and \( y, \) we consider the variables [5]

\[
\begin{align*}
x_s &= r_s \exp(i\theta_s), \quad \bar{x}_s = r_s \exp(-i\theta_s), \\
y_k &= \rho_k \exp(i\varphi_k), \quad \bar{y}_k = \rho_k \exp(-i\varphi_k),
\end{align*}
\]

(3)

Hence,

\[
\begin{align*}
r_s &= x_s \exp(-i\theta_s), \quad r_s = \bar{x}_s \exp(i\theta_s), \quad s = 1, 2, \ldots, n, \\
\rho_k &= y_k \exp(-i\varphi_k), \quad \rho_k = \bar{y}_k \exp(i\varphi_k), \quad k = 1, 2, \ldots, m,
\end{align*}
\]

and, consequently,

\[
\begin{align*}
\frac{dr_s}{dt} &= \frac{1}{2} \left( \frac{dx_s}{dt} e^{-i\theta_s} + \frac{d\bar{x}_s}{dt} e^{i\theta_s} \right), \quad s = 1, 2, \ldots, n, \\
\frac{d\rho_k}{dt} &= \frac{1}{2} \left( \frac{dy_k}{dt} e^{-i\varphi_k} + \frac{d\bar{y}_k}{dt} e^{i\varphi_k} \right), \quad k = 1, 2, \ldots, m.
\end{align*}
\]

In view of the system of equations (2), we obtain

\[
\begin{align*}
\frac{dr_s}{dt} &= \text{Re} \lambda_s r_s + \frac{1}{2} \left( f_{1s} e^{-i\theta_s} + \bar{f}_{1s} e^{i\theta_s} \right), \quad s = 1, 2, \ldots, n, \\
\frac{d\rho_k}{dt} &= \text{Re} \beta_k \rho_k + \frac{1}{2} \left( f_{2k} e^{-i\varphi_k} + \bar{f}_{2k} e^{i\varphi_k} \right), \quad k = 1, 2, \ldots, m.
\end{align*}
\]

(4)

In systems (4), the equations that correspond to complex-conjugate roots will repeat because the corresponding variables have equal moduli. Therefore, the number of different equations in systems (4) will be \( n_1 < n \) and \( m_1 < m, \) respectively.

Remark 1. If, in system (2), one represents the functions \( f_1 \) and \( f_2 \) in the form