Least squares estimators in measurement error models under the balanced loss function

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Abstract

The ultrastructural form of the measurement error model is considered and a comparison of the direct and the reverse regression estimators is made under the balanced loss function, which explicitly takes into account the accuracy of the predictions and the precision of estimation.

Key Words: Balanced loss function, direct and reverse regression, measurement errors, ultrastructural model.

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1 Introduction

There are two popular strategies for least squares estimation of the parameters in a linear regression relationship involving only two variables. One is the direct regression method based on the regression of the study variable on the explanatory variable, and the other is the reverse regression method based on the regression of the explanatory variable on the study variable. Considering the regression coefficient, the direct regression estimator is not only unbiased but also has superior performance, at least asymptotically, than the reverse regression estimator with respect to its mean squared error and the goodness of model fit, provided that there are no measurement errors in the observations. When the observations are contaminated by measurement errors, there is a dramatic change in the performance properties of both the estimators; see Cheng and Van Ness (1999) and Fuller (1987) for an interesting account.

In a seminal article, Zellner (1994) has recommended that the efficiency of any estimator should be examined by both the precision of estimation

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and the goodness of model fit which essentially reflects the accuracy of predictions. Accordingly, he has proposed the use of balanced loss function that succeeds in taking account of both criteria; see, e.g., Giles, Giles and Ohtani (1996), Ohtani (1998) and Wan (1994) for some interesting applications. Utilizing such a loss function, we compare the direct and reverse regression estimators in the context of the linear ultrastructural model considered by Dolby (1976).

2 The Main Result

Consider a set of \( n \) observations \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) on the explanatory variable \( X \) and study variable \( Y \). They are assumed to be error-ridden so that we can write

\[
\begin{align*}
y_i &= Y_i + u_i \quad (i = 1, 2, \ldots, n) \\
x_i &= X_i + v_i
\end{align*}
\]

where \( Y_i \) and \( X_i \) denote the true but unavailable counterparts while \( u_i \) and \( v_i \) are the measurement errors.

It is assumed that the errors \( u_1, u_2, \ldots, u_n \) are independently and identically distributed with mean 0 and variance \( \sigma_u^2 \). Similarly, the errors \( v_1, v_2, \ldots, v_n \) are independently and identically distributed with mean 0 and variance \( \sigma_v^2 \). We also assume that \( X_1, X_2, \ldots, X_n \) are random variables with means \( m_1, m_2, \ldots, m_n \) but the same variance \( \sigma^2 \). Finally, it is assumed that all these quantities are mutually independent.

This completes the specification of a linear ultrastructural model; see Dolby (1976). It reduces to the structural form of the linear measurement error model when \( m_1, m_2, \ldots, m_n \) are all equal. When \( X_1, X_2, \ldots, X_n \) are assumed to be nonstochastic so that \( \sigma^2 = 0 \), we obtain the functional form of the linear measurement error model. Lastly, it becomes the classical linear model, free from measurement errors, when \( \sigma_v^2 \) and \( \sigma^2 \) are both equal to 0.

If \( \beta \) denotes the slope parameter in the linear regression relationship of \( Y \) on \( X \), an application of the least squares procedure provides the following direct regression estimator of \( \beta \):

\[
b_D = \frac{s_{xy}}{s_{xx}},
\]

(2.2)