Bootstrapping forecast intervals in ARCH models

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Abstract

In this paper we develop a bootstrap method for the construction of prediction intervals for an ARMA model when its innovations are an autoregressive conditional heteroscedastic process. We give a proof of the validity of the proposed bootstrap for this process. For this purpose we prove the convergence to zero in probability of the Mallows metric between the empirical distribution function and the theoretical distribution function of the residuals. The potential of the proposed method is assessed through a simulation study.

Key Words: ARCH models, bootstrap method, prediction intervals.

AMS subject classification: 62M20, 62G09, 62E25

1 Introduction

Amongst the different objectives of applied statistical analysis, one of the most interesting when studying time series is forecasting. In this context, the methodology presented here is an application of the bootstrap procedure to the problem of interval forecasting for ARMA(p,q)-ARCH(r) time series.

The ARCH class of models was originally introduced by Engle (1982) to represent empirical evidence in those series in which uncertainty plays an essential role, as a convenient way of modelling time-dependent conditional heteroscedasticity.

Engle and Kraft (1983) were the first to consider the effect of ARCH on forecasting. They derived expressions for the multi-step prediction error

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variance in ARMA models with ARCH errors, but did not discuss the characteristics of the prediction error distribution. Baillie and Bollerslev (1992) found ex ante prediction confidence intervals by using parametric hypotheses on the conditional error distribution. However, these conditions are usually not satisfied, and forecast intervals typically depend upon an assumption on the conditional error distribution. In addition, the standardized prediction error distribution depends nontrivially on the information set at time $t$. This complicates the expressions for the higher-order conditional moments, and consequently the construction of the forecast intervals.

Whilst the majority of empirical studies using ARCH models tend to rely on parametric specifications, we propose a bootstrap procedure when the assumptions on the conditional error distribution may not be justified. Efron’s (1979) bootstrap provides a non-parametric method of estimating distributions with small sample sizes, when derivations of the sampling distribution are analytically intractable. In particular, we develop a bootstrap method for estimating the conditional distribution of $t + s$, given the observations up to time $t$. The study of the bootstrap for time series and dynamic regression models was begun by Freedman (1981). Thombs and Schucany (1990) give bootstrap prediction intervals for AR models, while McCullough (1994) implements the bias-correction bootstrap to real data. Kreiss and Franke (1992) prove the asymptotic validity of the bootstrap applied to M-estimators of stationary ARMA models.

In this paper we use a recent bootstrap procedure for constructing prediction confidence intervals, which does not require any additional hypotheses on the conditional error distribution. It is based on the works of Febrero et al. (1995) and Cao et al. (1995), and is designed to mimic the distribution of the forward values, conditional on all previous values of the series. We will only resample ahead conditional on the estimates of the model parameters, since the ARCH models are “ad hoc” methods for measuring shifts in the variance over time.

The proposed method does not require a knowledge of the backward residuals which are difficult to estimate in dynamic models with time-dependent conditional variances, as several parametric specifications of the conditional variance function can be evaluated. Consequently, no single parametric specification of the conditional density appears to be suitable for all conditionally heteroscedastic data. Moreover, it is computationally faster than the method proposed by Thombs and Schucany (1990) and ex-