FLOW AND STABILITY OF A FREE AXISYMMETRIC FILM

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Axisymmetric free film flows are encountered in many technological processes [1] associated with the atomization of liquids. The transversely symmetric flow produced by an atomizer consisting of two coaxial disks rotating at the same speed and arranged to form an annular gap is considered.

1. Let a viscous incompressible liquid be introduced at a constant flow rate $Q$ near the center of the gap and be ejected into the ambient medium by the action of the centrifugal force. The motion of the liquid in the film thus formed is described by the system of Navier-Stokes equations, and the motion of the ambient gas, assumed to be incompressible, by the system of Euler equations; on the lower and upper boundaries of the film the kinematic conditions for the liquid and the gas, the conditions of continuity of the normal stresses with allowance for surface tension, and the zero shear stress conditions for the liquid are satisfied; it is assumed that at a sufficient distance from the film the gas is at rest [2].

In order to describe the motion of the liquid and the gas we will use the cylindrical coordinate system $r, \theta, z$ moving with the axis of rotation of the gap, whose center corresponds to the value $z = 0$. The pressure in the liquid or gas $p_f$ and the velocity components $u_r, u_\theta, u_z$ are represented in the form:

$$p_f = p_0 + \rho \Omega H_c \rho, \quad u_r = \Omega rv, \quad u_\theta = \Omega rv, \quad u_z = \Omega H_c w$$

where $\rho$ is the density of the liquid, and $\Omega$ and $H_c$ are the angular velocity and the width of the gap. As the independent variables we will employ the quantities $x = \ln (r/R), \theta, y = z/H_c$, and $s = \Omega t$, where $t$ is time and $R$ is the radius of the gap.

For a relatively thin film the problem of determining its steady flow can be separated from the consideration of the motion of the gas. By analogy with film flow over the surface of a rotating disk [2, 3], for a thin free film, without allowance for terms of the order of $(H_c/r)^2$ and surface tension, in the case of a flow possessing axial symmetry and symmetry about the plane $y = 0$ the equations and boundary conditions have the form:

$$\frac{\partial u}{\partial x} + 2u + \frac{\partial v}{\partial y} = 0$$

(1.1)

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial y} + u^2 - v^2 = \frac{1}{Re} \frac{\partial^2 u}{\partial y^2}$$

(1.2)

$$u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial y} + 2uv = \frac{1}{Re} \frac{\partial^2 v}{\partial y^2}$$

(1.3)
where $h$ is the surface of the film; $Re = \frac{\Omega R^2}{\nu}$, and $\nu$ is the kinematic viscosity coefficient. Here, (1.1) is the continuity equation; (1.2) and (1.3) are the equations of motion for the radial and azimuthal velocity components, respectively; (1.4) is the kinematic condition and the conditions of zero shear stress on the film boundary; and (1.5) are the symmetry conditions.

The formulation of the problem of the motion of the film also includes the initial conditions

$$x=0: \ h=0.5, \ u=U_0(y), \ v=V_0(y) \tag{1.6}$$

where $U_0$ and $V_0$ are given functions.

For the numerical solution of the problem (1.1)--(1.6) we will use a colocation method [2--4] in which we introduce the stream surfaces $h_n(x), n = 1, \ldots, N$, where $h_1 \equiv 0, \ h_N \equiv h$, and the values of the velocity components on them $u_n(x) = u(x, h_n(x)), \ v_n(x) = v(x, h_n(x))$. From (1.1)--(1.5) there follows a system of ordinary differential equations for $h_n, u_n, \text{ and } v_n$ containing the values of the second derivatives of the velocity components with respect to $y$ on the stream surfaces. In order to calculate them a Chebyshev polynomial approximation of the velocity components that takes into account boundary conditions (1.4) and (1.5) is used.

From (1.1), (1.4), and (1.5) and the flow symmetry it follows that

$$q(x) = \frac{Q}{2\pi^2 \Omega H}. \tag{1.7}$$

Calculations show that irrespective of the form of the initial profiles of the velocity components, with distance from the edge of the gap the flow becomes uniform and independent of $y$. Below we present the results of the calculations for initial profiles of the form:

$$U(x) = 1 + \frac{4}{11}q(0)(30y^4 - 15y^2 + 1.975), \quad V(x) = 1 + \frac{4}{11}q(0)(\frac{1}{2}y^2 - y^4 - \frac{1}{16}). \tag{1.8}$$

where $a$ is a parameter. The choice of (1.7) is associated with the fact that these profiles, which are polynomials of the lowest order satisfying (1.4) and (1.5), qualitatively correspond to the solution of the problem of radial flow in a rotating annular gap [5, 6]; moreover, for (1.7) it is possible to study the parametric relations.

Figure 1 gives examples of the variation of the values, averaged over the thickness of the film, of the radial and azimuthal velocity components $u_m$ (curves 1 and 4) and $v_m$ (curves 2 and 5), respectively, and of the film thickness $h$ (curves 3 and 6) on the interval of transition to uniform flow for $Re = 100, \ a = 15$ in the cases $q(0) = 1$ (curves 1--3) and 1.8 (curves 4--6). The form of the solution corresponding to $q(0) = 1$ is presented in Fig. 2, where curves 1 to 3 relate to $U_m = u/u_m$ and curves 4 to 6 to $V_m = v/v_m$, curves 1 and 4 corresponding to $x = 0$, curves 2 and 5 to $x = 0.4$, and curve 3 and 6 to $x = 0.88$.

The length of the interval of transition to uniform flow $x_a$ and the form of the