ESTIMATING SEASONAL DEFORMATIONS IN THE EARTH’S CRUST IN CORRECTING FOR EFFECTS ON THE DETERMINATION OF THE EARTH’S ROTATIONAL PARAMETERS

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Estimates are made of the vertical deformations in the Earth’s crust produced by variable loads on regions having characteristic dimensions of the order of thousands of kilometers, which arise from the development and displacement of cyclones and anticyclones and also from the accumulation of snow and the passage of floods on large rivers. It is found that the total deformation in Siberia can attain 30 cm. It is necessary to correct for the periodic displacements in observational points in the service for determining the Earth’s rotational parameters. Suggestions are made on organizing experiments on the deformations.

The stability in the positions of the points defining the terrestrial coordinate system TCS has a substantial effect on the accuracy in determining the Earth’s rotation parameters ERP as elements in the orientation of that coordinate system relative to the inertial with the contemporary accuracy in observational facilities.

Geodynamic observations are used to determine the Earth’s rotation parameters, but corrections are applied only for the regular vertical movements of the points on account of lunar–solar tides, and the same applies to research on contemporary crustal movements. All other displacements are perceived as errors of measurement. On the other hand, the vertical deformations considered in this paper exceed the instrumental errors of the means of measurement by about an order of magnitude.

There are considerable random fluctuations (up to 2 msec in universal time [1]) in the Earth’s rotation after those factors have been excluded: the constant tidal retardation and the effect from changes in the atmospheric angular momentum, and these have not yet been satisfactorily explained in geodynamic models. Filtering the measurements to remove these deformations in the observational network enables one to reduce the noise component in the fluctuations of the Earth’s rotational rate by about an order of magnitude. It is thus possible to examine more detailed effect from the influencing factors.

Here we develop a theory and experimental methods for determining vertical displacements of points on the Earth’s surface due to time-varying loads from atmospheric, snow, and water masses, and we also examine the effects of those displacements on ERP observations (variations in the angular velocity and polar coordinates).

It is assumed that the crust is deformed elastically by variable brief loads on regions with characteristic dimensions of the order of thousands of kilometers, which arise from the development and displacement of cyclones and anticyclones, from the accumulation of snow, and also the passage of snow and rain floods in large rivers. The additional crustal load produces vertical and horizontal displacements. Here we consider only the vertical deflections. The theoretical estimates have been made from the Boussinesq treatment [2] for a semi-infinite planar elastic medium loaded by a distributed pressure \( q(x, y) \). We neglect the mass forces (gravitational attraction) and the effects of the Earth’s surface curvature to a first approximation. Those simplifications allow one to obtain exact solutions, which give an indication of the order of the effects.

The vertical displacements of a plate surface in the Boussinesq treatment are given by

\[
W(x, y) = \frac{1}{\pi} \int \int \frac{q(x', y') \, dx' \, dy'}{\sqrt{(x-x')^2 + (y-y')^2}},
\]

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in which $x$ and $y$ are the coordinates of the point at which the deformation is determined, $x'$ and $y'$ are the coordinates of the current point, $\Theta = (1 - \sigma^2)/\pi E$, $\sigma$ is Poisson's ratio, and $E$ is Young's modulus.

We now consider some particular cases. Let a region bounded by a circle of radius $R$ in the crust be subject to a constant load

$$ q = \pm \rho_0 = \text{const}. $$

With some idealization, this represents the situation under a large water basin when the surface level alters. The exact solution for the region within the circle ($r < R$) gives

$$ W_1(r) = \pm 4\Theta \rho_0 R E(r/R), $$

in which $E(r/R)$ is a complete elliptic integral of the second kind.

Outside the loaded region ($r > R$) we get

$$ W_2(r) = \pm 4\Theta \rho_0 \frac{r}{R} \left[ E \left( \frac{R}{r} \right) - \left( 1 - \frac{R^2}{r^2} \right) K \left( \frac{R}{r} \right) \right], $$

in which $K(R/r)$ is a complete elliptic integral of the first kind.

From (2) and (3) we get

$$ W(0) = \pm 2\pi \rho_0 R \quad \text{for} \quad r = 0; $$

$$ W(R) = \pm 4\Theta \rho_0 R \quad \text{for} \quad r = R; $$

$$ W(r) \approx \pm \pi \rho_0 \frac{R}{r} \quad \text{for} \quad r \gg R. $$

If the constant load acts within a region bounded by a rectangle having sides $x_0$ and $y_0$ ($y_0 > x_0$), then the deformation at the center is

$$ W_e = 2\Theta \rho_0 \left( x_0 \arsh \frac{k}{x_0} + y_0 \arsh \frac{k}{y_0} \right). $$

The deformations at the mid-points of the long sides of the rectangle (at the shores of an elongated body of water) are

$$ W_s = \Theta \rho_0 \left( x_0 \arsh \frac{2y_0}{x_0} + 2y_0 \arsh \frac{x_0}{2y_0} \right). $$

We now consider a model having a Gaussian pressure distribution, which corresponds approximately to the situation where an anticyclone (+) or a cyclone (-) lies above the observation point:

$$ q(x, y) = \pm \rho_0 \exp \left( -\frac{x^2 + y^2}{2s^2} \right), $$

in which $s$ is the radius of the region, at the boundary of which $p = 0.606 \rho_0$, and $\rho_0$ is the pressure at the center.

Integration in (1) gives

$$ W = \pm \pi \sqrt{2\pi} \Theta \rho_0 s I_0 \left( \frac{r^2}{4s^2} \right) \exp \left( -\frac{r^2}{4s^2} \right), $$

in which $I_0$ is a modified Bessel function.