PARAMETRIC IDENTIFICATION PROBLEMS

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An approach to the generation of stopping rules in parametric identification problems is proposed on the basis of the computation of a statistic of the difference between two successive estimates. This statistic is also used for fault detection in the Kalman filter.

The identification of dynamic objects described by difference equations of a known order, but with unknown coefficients, entails the estimation of these coefficients, i.e., it is a parametric problem. It is always attended by the problem of determining the stopping time of the computations or what essentially amounts to the verification of sufficiency of the number of observations when a prescribed accuracy is attained on the part of the computed estimates.

The results of the general mathematical theory of optimal stopping rules [1-5], a latter-day branch of probability theory, have not enjoyed any appreciable application in the generation of stopping rules in parametric identification problems. This situation is attributable to the complexity of adapting various statistical tests of a general nature to real applied problems and algorithms.

The application of the rules proposed in [6, 7] for stopping of the identification process runs into several difficulties, one of which is the need to specify an admissible error ellipsoid or an admissible measure of this ellipsoid.

In this article we propose a stopping rule that is free of these shortcomings; it is based on the comparison of a statistic of the difference between two successive estimates with a predetermined confidence limit of the chi-square distribution. The indicated statistic is also used for fault detection in the Kalman filter.

GENERATION OF STOPPING RULES

We introduce the following stopping rule in application to multidimensional parametric identification problems:

\[ r_2^2 = \left( \hat{\theta}_i - \hat{\theta}_{i-1} \right)^T D_{\Delta \theta_i}^{-1} \left( \hat{\theta}_i - \hat{\theta}_{i-1} \right) < \varepsilon, \]  

(1)

where \( D_{\Delta \theta_i} \) is the covariance matrix of the discrepancy between two successive estimates \( \hat{\theta}_i \) and \( \hat{\theta}_{i-1} \), and \( \varepsilon \) is a predetermined small number.

We assume that the well-developed theory of Kalman filtering is used to estimate the parameters from a sequence of observations with Gaussian tolerances of the measurement errors and system noise. In this situation the Kalman filter yields an estimate with expected value equal to the estimated quantity and a Gaussian distribution function. The discrepancy \( \hat{\theta}_i - \hat{\theta}_{i-1} \) then has a normal distribution as well, since it is a linear combination of two Gaussian random variables [8]. With these considerations in mind we know that the statistic \( r_2^2 \) has a \( \chi^2 \) distribution with \( n \) degrees of freedom (\( n \) is the number of dimensions of the vector \( \theta \)), and the threshold values of \( r_2^2 \) can be found by determining the tabulated values of the \( \chi^2 \) distribution for a given level of significance.

It is evident from relation (1) that the smaller the value of \( r_2^2 \), the greater will be the consistency of the estimates. Usually in the testing of consistency in such cases the lower limit of the confidence interval must be equal to zero, and the upper limit is determined by the level of significance \( \alpha_1 \).
To test the consistency of the estimates, we adopt the level of significance $\alpha_1$, which corresponds to the confidence coefficient $\beta_1 = 1 - \alpha_1$. We specify the threshold $\chi^2_\beta$ in terms of this probability, using the distribution of the investigated statistic $r^2$:

$$p(\chi^2 < \chi^2_{\beta}) = \alpha_1, \quad 0 < \beta_1 < 1.$$ 

We stop the estimation process when $r^2 < \chi^2_\beta$, since further observations yield insignificant improvement of the identified model and are deemed impractical in this event. If the quadratic form $r^2$ is larger than or equal to the specified threshold $\chi^2_{\beta}$, estimation should be continued.

This stopping rule can be used to make a timely decision to stop the estimation process in the identification of dynamic systems, and it does not require large computational expenditures.

**FAULT DETECTION IN THE KALMAN FILTER**

The chi-square test discussed above can also be used to troubleshoot the Kalman filter. The statistic $r^2$ must be compared with the confidence limit of the $\chi^2$ distribution determined from the expression

$$p(\chi^2 < \chi^2_{\beta}) = \beta_1, \quad 0 < \beta_1 < 1,$$

and a decision must be made on the basis of the rule

$$\begin{cases} r^2 \leq \chi^2_{\beta}, & \text{the Kalman filter is operating normally;} \\ r^2 > \chi^2_{\beta}, & \text{faults are present.} \end{cases}$$

Consequently, by comparing the above-defined statistic $r^2$ with the confidence limits obtained for the corresponding $\chi^2$ distribution it is possible to solve two problems at once: to determine the stopping time of the identification process and to detect faults in the Kalman filter in due time.

Figure 1 shows a graph of the probability density function of the $\chi^2$ distribution with $n = 4$ degrees of freedom and the computed confidence limits for $\beta_1 = 1 - \alpha_1 = 0.2$ and $\beta_2 = 1 - \alpha_2 = 0.95$ ($\alpha_2$ is the level of significance), where the numerals indicate: 1) the zone of stopping of observations; 2) the zone of estimation; 3) the fault-detection zone.

It is evident from Fig. 1 that the domain of potential application of the Kalman filter is partitioned into three zones. Estimation is assumed to continue as the value of the statistic $r^2$ is determined between the confidence limits $\chi^2_{\beta_1}$ and $\chi^2_{\beta_2}$. If $r^2 < \chi^2_{\beta_1}$, the estimation process should be stopped, since further observations are assumed to yield insignificant improvement of the...