RADIO MEASUREMENTS

INVESTIGATION OF THE POTENTIAL QUALITY OF MEASUREMENTS OF THE ELECTRICAL PARAMETERS OF A PLANE SURFACE USING DATA ON ITS NATURAL RADIO-THERMAL RADIATION

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The results of investigations of the behavior of the maximum errors in measuring the electrical parameters of a plane interface between two media, defined by the diagonal elements of the covariance matrices of the errors, inverse to the Fisher information matrices, using data on the horizontally and vertically polarized thermal radiation are presented. The results of computer statistical modeling of quasi-optimal algorithms of estimates of these parameters are also given.

When solving problems of the remote sensing of the Earth’s surface by measuring the electrical parameters of plane specimens of different materials under laboratory conditions, it is extremely important to choose the directions in which their natural radiation is received so as to reduce the measurement errors to a minimum. A considerable number of publications [1-5] are devoted to the solution of inverse problems of establishing the values of the electrical parameters of plane interfaces between two media. However, in the majority of these no information is given on the assumed behavior of the measurement errors, which makes it difficult to set up experiments and to choose the appropriate conditions for carrying them out. This information can be obtained from calculations of the potential accuracies (the maximum measurement errors), defined by the Rao—Cramer inequality or the diagonal elements of the covariance matrices of the errors, inverse to the Fisher information matrices [6]. An exception to this is the paper by Volosyuk and Kravchenko [7], but they analyze the potential accuracies of the measurements of the parameters of a plane surface only partially in order to illustrate the general theory of optimum estimates of the parameters of arbitrary surfaces using the results of recordings of natural radiation.

In this paper we present more complete and useful practical results of investigations of the behavior of the maximum errors when measuring the parameters of a plane interface between two media (one of which is air with a magnetic permeability and permittivity of unity) by receiving natural radiation in two polarizations (separately or together) when measuring one or two unknown parameters when the remaining ones are known.

The complete set of measured parameters are the permittivity $\varepsilon$, the thermodynamic temperature $T_0$, and the direction of the radiation characterizing the unknown slope of the surface and defined by the angle $\theta$ between the normal to the surface and the line joining the center of the part being investigated to the phase center of the receiving antenna of the radiometer.

We will present some initial relations [7].

1. The observation equation

$$U_m(t) = U_{m_0}(t, \lambda) + n_m(t), \ t \in (0, T), \ \lambda = (\varepsilon, \theta, T_0),$$

where $U_m(t, \lambda)$ are the oscillations of the output of the linear radiometer channel due to radiation from the surface being investigated and the external background (to investigate the effect on the error in measuring the parameters of the surface itself directly, we neglect the radiation due to external illumination), $n_m(t)$ is internal (white) noise at the output of the linear channel of the receiver (it is negligibly small and mainly plays the role of a regularizing correction when solving the inverse problem.

of estimating the parameters), and the subscript \( m = (v, h) \) represents the form of polarization \( (v \) represents vertical polarization and \( h \) represents horizontal polarization).

2. The optimum algorithm using the maximum-likelihood method (system of equations) for estimates of the parameters \( \bar{x} \)

\[
\int_{-\infty}^{\infty} \frac{\left| \hat{K}_m(j2\pi f) \right|^2}{G_{mZ}(f, \bar{x})} df = \int_{-\infty}^{\infty} \frac{\left| \hat{K}_m(j2\pi f) \right|^2}{G_{mZ}(f, \bar{x})} \frac{|S_{mT}(j2\pi f)|^2}{T} df,
\]

where

\[
G_{mZ}(f, \bar{x}) = |\hat{K}_m(j2\pi f)|^2 B_{Am}(f, \bar{x}) + N_{am}/2;
\]

\[
B_{Am}(f, \bar{x}) = 0.25 \int \sigma_B(f, \bar{x}, \theta) F(\theta - \theta_0) d\theta;
\]

\( B_m(\cdot) \) is the spectral brightness of the surface in question, \( F(\cdot) \) is the power radiation pattern of the receiving antenna of the radiometer, and \( \hat{K}(2\pi f) \) is the transfer constant of the linear channel of the radiometer with center tuning frequency \( f_0 \).

We will assume that the angle \( \theta_0 \) corresponding to the direction of the maximum value of the radiation pattern varies in the plane in which the normal to the surface is situated and in which the radiation propagates. In this case, without loss of generality, we can consider the plane problem and the integration need only be carried out with respect to the single variable \( \theta \).

3. The quasioptimal algorithm

\[
B_{Am}(f_0, \bar{x}) = (\Delta f)^{-1} \int_{-\infty}^{\infty} |\hat{S}_{mT}(j2\pi f)|^2 df =
\]

\[
= (\Delta f)^{-1} \int_0^T U_{mZ}(t) dt,
\]

where

\[
\Delta f_m = \int_{-\infty}^{\infty} |\hat{K}_m(j2\pi f)|^2 df.
\]

Henceforth we will assume that \( \Delta f_m = \Delta f \), and that both channels for receiving the vertically and horizontally polarized radiation are identical.

4. The antenna temperature \( T_{Am} \) and the radio-brightness temperature of the surface \( T_{bm} \) are related to the corresponding spectral brightnesses by the Rayleigh–Jeans formula [2]

\[
B_{Am} = kT_{Am}/c^2, \quad B_m = kT_{bm}/c^2,
\]

where \( k \) is Boltzmann’s constant.

For a plane surface

\[
T_{bm} = (1 - |K_{FV}(e, \theta_0)|^2) T_0;
\]

\[
K_{FV} = \frac{e \cos \theta_0 - V e - \sin \theta_0}{e \cos \theta_0 + V e - \sin \theta_0},
\]

\[
K_{FT} = \frac{\cos \theta_0 - V e - \sin \theta_0}{\cos \theta_0 + V e - \sin \theta_0},
\]

are the Fresnel reflection coefficients.

5. The maximum errors of the measurements are determined by the diagonal elements of the covariance matrix of the errors \( \Phi^{-1} \), inverse to the Fisher information matrix

\[
\Phi_{kk} = \frac{T_{Am}^2}{2} \sum_{m=1}^{2} \frac{\partial \ln T_{Am}(f_0, \bar{x})}{\partial \lambda_m} \frac{\partial \ln T_{Am}(f_0, \bar{x})}{\partial \lambda_m}.
\]