PHOTON RADIATION OF AN EXTENDED RELATIVISTIC OBJECT

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The operator of the electromagnetic interaction of an extended relativistic object is constructed. It is shown that the main difficulty, which arises due to noncommutativity of the coordinates and velocities of the charges in relativistic quantum mechanics, may be overcome by a special ordering of the operators.

1. Introduction

The problem of photon emission by composite systems in nonrelativistic quantum mechanics is described in numerous textbooks (see, e.g., [1]). Its solution leads to the construction of the electromagnetic current, which, in turn, is defined by the coordinates and velocities (momenta) of the electric charges. This current defines the full variety of electromagnetic properties of atoms and nuclei. A similar approach is also valid in hadron physics, but only for a heavy quarkonium. The reason for this constraint is simple: we know very little about the quantum (or even classical) theory of relativistic extended systems, i.e., systems that are different from point particles. On the other hand, even in our present knowledge, the problem of constructing an electric current is very nontrivial. Indeed, according to the "no-interaction" theorem of Jordan, Currie, and Sudarshan [2], the coordinates of interacting particles (charges) cannot be canonical even in classical mechanics, which, in turn, makes them noncommuting operators in quantum theory. The naive prescription for the electromagnetic interaction,

\[ H_{\text{int}} = j^\mu(x)A_\mu(x), \]

becomes very indeterminate, as, here, we encounter the problem of defining the function of noncommuting variables and, at the same time, preserving the gradient invariance of the interaction.

In this paper, we present a solution of this interesting problem in the case of the simplest extended relativistic system that admits the interpretation as a system of two interacting particles [3]. This system—the straight-line (rigid) string—was used in different approaches [4, 5, 6] as the basis for a rather realistic model of light mesons. This system is advantageous because we know its geometrical content from the very beginning, i.e., the coordinates of the endpoints of the string, which we identify with the coordinates of the charges. It follows from the string dynamics that the velocities of the string endpoints never leave the light cone, i.e., the causality principle holds true.

2. Straight-line string

As mentioned in the Introduction, there are different approaches to the theory of the straight-line string. Here, we briefly describe the clearest way of reducing the general Nambu–Goto string. This string is described by the classical action, which is proportional to the area of the world sheet,

\[ A = \frac{1}{2\pi\alpha'} \int d\tau \int_0^\pi d\sigma \sqrt{\dot{x}^2 - \dot{x}^2 x'^2}, \]  

Equation (1)

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where $\alpha'$ is the coefficient with the dimension $(\text{length})^2$, $x_\mu(\sigma, \tau)$ are the coordinates of the world sheet, and the dot and prime denote the derivatives with respect to $\tau$ and $\sigma$, respectively. The straight-line configurations of the string are parameterized by the following ansatz:

$$x_\mu(\sigma, \tau) = X_\mu(\tau) + q_\mu(\tau)\varphi(\sigma, \tau).$$

Note that the endpoints of the string, $x^{1,2}_\mu(\tau)$, correspond to $\sigma = 0$ and $\sigma = \pi$ in (2). Therefore,

$$x^{1,2}_\mu(\tau) = X_\mu(\tau) + q_\mu(\tau)\varphi^{1,2}(\tau)$$

with

$$\varphi^{1,2}(\tau) = \varphi(\sigma, \tau)|_{\sigma=0,\pi}.$$  

Substituting (2) into general action (1), we obtain

$$A = \frac{1}{2\pi\alpha'} \int d\tau \int_{\varphi_1(\tau)}^{\varphi_2(\tau)} d\varphi \sqrt{(-\dot{q}^2)(\dot{X}_\perp + \dot{q}_\perp \varphi)^2},$$

where we denote $a_{\mu\perp} \equiv a_\mu - \frac{\partial^2}{\partial \varphi^2}$. In this reduced system, the dynamic variables are $X_\mu(\tau), q_\mu(\tau)$, and $\varphi^{1,2}(\tau)$. One can find $\varphi^{1,2}(\tau)$ by using the equation of motion

$$\frac{\delta A}{\delta \varphi^{1,2}(\tau)} = 0,$$

whence,

$$\varphi^{1,2}(\tau) = -\frac{\dot{X}_\perp \dot{q}_\perp}{\dot{q}_\perp^2} \pm \left(\frac{\dot{X}_\perp^2}{\dot{q}_\perp^2} - \frac{(\dot{X}_\perp \dot{q}_\perp)^2}{\dot{q}_\perp^2}\right)^{1/2}.$$  

Integrating (4) w.r.t. $\varphi$, we obtain the following action for the straight-line string:

$$A = \frac{1}{4\alpha'} \int d\tau (q^2 \dot{q}_\perp^2)^{1/2} \left[\frac{\dot{X}_\perp^2}{\dot{q}_\perp^2} - \frac{(\dot{X}_\perp \dot{q}_\perp)^2}{\dot{q}_\perp^2}\right].$$

Here, only $X_\mu(\tau)$ and $q_\mu(\tau)$ are dynamic variables. Action (6) is invariant w.r.t. the following gauge transformation:

$$\tau \to \varphi(\tau),$$

$$X_\mu \to X_\mu(\tau) + \alpha(\tau)q_\mu(\tau),$$

$$q_\mu(\tau) \to \beta(\tau)q_\mu(\tau),$$

which is the remaining subgroup of the initial reparameterization gauge group.

The transition to the Hamiltonian formalism is straightforward. There are two canonical momenta, $P_\mu$ and $p_\mu$, conjugate to $X_\mu$ and $q_\mu$, respectively,

$$\{X_\mu, P_\nu\} = \{q_\mu, x_\nu\} = g_{\mu\nu},$$

and there are three first-class constraints due to the three gauge transformations (7).

$$\Phi_0 = \alpha' P^2 + (q^2 \dot{q}_\perp^2)^{1/2} \approx 0, \quad p^\mu = p_\mu - P_\mu \frac{DP}{P^2} \approx 0,$$

$$\Phi_1 = Pq \approx 0, \quad \Phi_2 = pq \approx 0.$$