ABSTRACT - The linear integral equation of convolution type of Sobolev's function \( \phi \) is solved computationally using a new method for the numerical inversion of Laplace transforms.

I. Introduction.

In the Sobolev theory of radiative transfer in a plane-parallel atmosphere [1] a fundamental role is played by the function \( \Phi(\tau), 0 \leq \tau < \infty \). This function is defined by the integral equation of convolution type

\[
\Phi(\tau) = K(\tau) + \int_0^\tau K(\tau - \tau') \Phi(\tau') d\tau'.
\]
The auxiliary function $K(r)$ is defined to be

$$K(r) = \frac{\lambda}{2} \int_0^1 e^{-r\eta} \varphi(\eta) \, d\eta,$$

where $\lambda$ is the albedo for single scattering, and $\varphi(\eta)$ is the Ambarzumian-Chandrasekhar [2,3] function that satisfies the nonlinear integral equation

$$\varphi(\eta) = 1 + \frac{\lambda}{2} \eta \varphi(\eta) \int_0^1 \frac{\varphi(\eta')}{\eta + \eta'} \, d\eta'.$$

If we denote the Laplace transforms of the functions $\Phi(r)$ and $K(r)$ by $F(s)$ and $G(s)$, respectively, we see that

$$F(s) = \frac{G(s)}{1 - G(s)},$$

and

$$G(s) = \frac{\lambda}{2} \int_0^1 \frac{\varphi(\eta)}{1 + \eta s} \, d\eta.$$

Here we wish to point out that we can quite easily obtain numerical values of the function $\Phi$ by making use of Eqs. (4) and (5) and an effective scheme for the numerical inversion of Laplace transforms.

II. Numerical inversion of Laplace transforms.

Let us first sketch the method of numerical inversion employed. Let

$$H(s) = \int_0^\infty e^{-st} h(t) \, dt.$$

We shift to a finite interval of integration via the transformation

$$e^{-t} = r.$$

Equation (6) becomes

$$H(s) = \int_0^1 r^{s-1} h(- \log r) \, dr.$$

Then we approximate the integral above, using a gaussian quadrature for-