The Extrusion Force and the Mean Strain Rate During the Extrusion of Strain Rate Sensitive Materials

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The force required to extrude a strain rate sensitive material under conditions of homogeneous deformation is calculated by the uniform work method. The force is shown to be a strong function of the rate sensitivity of the material. A new definition of the mean strain rate during extrusion is proposed, which is particularly appropriate to rate sensitive materials. For extrusion ratios of 40 to 160 and a power law strain rate sensitivity of 0.2, the present method leads to extrusion pressures 30 to 60 pct higher than when no account is taken of the rate sensitivity of the flow stress in the die zone. In an appendix, a design for a constant true strain rate (CTSR) die is described, which permits experiments to be carried out without the necessity for rate sensitivity corrections.

The force required for extrusion can be estimated by a number of methods, the most common of which are the Hencky slip line analysis, the Johnson and Kudo upper bound techniques, the Sachs slab method, and the Siebel uniform work method. Each technique has several limitations. For example, the slip line method is usually limited to the analysis of plane strain problems, and both the slip line and upper bound methods exclude the effects of work hardening and of strain rate on the local flow stress. Similarly, the slab and uniform work methods overlook frictional and internal shear losses, for which corrections have to be made. The latter two techniques do, however, permit the effects of both work hardening and of strain rate sensitivity to be taken into account; nevertheless, while corrections for the former are readily carried out, the strain rate sensitivity of the flow stress is generally neglected. It is the purpose of the present work to a) evaluate the pressure increase which results when a material is strain rate sensitive during flow through the deformation zone; and b) propose a new definition of the mean strain rate which is particularly appropriate to strain rate sensitive materials. In an appendix, a design for a constant true strain rate (CTSR) die will also be considered, which permits experiments to be carried out without the necessity for rate sensitivity corrections.

According to the uniform work method, the homogeneous work of extrusion is evaluated by considering the work done in deforming an element of material along a streamline of flow. The homogeneous work per unit volume, \( W_h \), is then given by: \[ W_h = \int_0^1 \sigma \varepsilon \ln R = P_h \] [1]

Here \( \sigma \) is the equivalent uniaxial flow stress of the material in the course of deformation, \( \varepsilon \) is the true strain, \( R \) is the extrusion ratio, \( \varepsilon = (1/\ln R) \int_0^\varepsilon \sigma \varepsilon \) is the mean flow stress, which takes into account the strain hardening occurring during deformation, and \( P_h \) is the extrusion pressure for homogeneous flow.

In the absence of friction and redundant work, an identical result is obtained by the slab method of analysis. In order to calculate extrusion pressures, the expression for \( P_h \) is easily computed for axisymmetric and other simple die geometries, while \( \varepsilon \) is usually obtained from independent tests performed by hot compression, tension, or torsion at suitable temperatures and strain rates.

When redundant work due to shearing is also taken into account, an empirical shear or efficiency factor \( C \approx 1.5 \) is frequently introduced, so that the increased extrusion pressure \( P_{h+\tau} \) is given by:

\[ P_{h+\tau} = C \varepsilon \ln R \] [2]

Eq. [2] can be further modified to allow for the effect of friction which, in direct extrusion, leads to a pressure contribution proportional to the length of unextruded billet.

The present analysis is limited to the consideration of extrusion under conditions of homogeneous deformation. Such conditions are approached when extrusion is carried out by the indirect process, so that there is no relative motion between billet and container, and when the flow in the die is well lubricated. An example of nearly perfect homogeneous flow is shown in Fig. 1 for the indirect extrusion of ice. In this case the die zone is lubricated by friction melting of the ice at the die/ice interface. The attainment of such simple loss conditions permits a relatively straightforward correction to be made for the strain rate sensitivity of the flow stress.

Consider the flow of a disc-shaped element of material through the die zone shown in Fig. 2. As before, the pressure required for flow is:

\[ P_h = \frac{F}{A_h} = \int_0^\varepsilon \sigma \varepsilon \] [3]

where \( F \) is the extrusion force and \( A_h = (\pi/4)D_0^2 \) is the cross-sectional area of the die entry. Previously, the flow stress \( \sigma \) was considered to be a function of strain \( \varepsilon \), and the strain rate dependence of \( \sigma \) was neglected. In the present case, the strain dependence of \( \sigma \) will be neglected, as materials can reach a rela-
tively steady state of flow stress during deformation at elevated temperatures; however, the strain rate dependence \( \sigma(\dot{\varepsilon}) \) will be taken into account instead. For this purpose, the strain rate profile for homogeneous flow through the die must be established, and the strain rate sensitivity of the material under the deformation conditions must also be known.

The former is readily derived from the geometry of the die, but very little information is usually available concerning the latter. As an approximation, it will therefore be assumed that the strain rate sensitivity of the material as it travels through the die is given by the power relations:

\[
\dot{\varepsilon}_s = K_0 \dot{\varepsilon}_s^n \quad \text{and} \quad \sigma_s = K' \dot{\varepsilon}_s^m
\]

Here \( \dot{\varepsilon}_s \) and \( \sigma_s \) are steady-state strain rates and flow stresses, \( K \) and \( K' \) are the steady-state strain rate and flow stress coefficients, respectively, \( m = (1/n) \) is the strain rate sensitivity of the flow stress, and \( K' = (1/K) m \). The exponents \( n \) and \( m \) are constant over moderate strain rate and temperature ranges; \( K \) and \( K' \), however, are sensitive functions of temperature. \( K \) and \( n \) are generally determined in secondary creep tests, and the inverse coefficients \( K' \) and \( m \) in steady-state hot working tests. The exponent \( n \) can be expected to be slightly higher for transient than for uniform conditions, but this difference will be neglected in the present analysis.

By combining Eqs. [3] and [4], the extrusion force can be expressed as:

\[
F = A_0 K' \int_0^R (\dot{\varepsilon}_s)^m d\varepsilon
\]

where the local strain rate \( \dot{\varepsilon}_s \) is given by

\[
\dot{\varepsilon}_s = \frac{4 V_0 D_0^2 \tan \theta}{D^3}
\]

Here \( V_0 \) is the velocity of the entering material (velocity of the ram), \( D_0 \) and \( D \) are the initial and local diameters, and \( \tan \theta \) is the local die semi-angle. For the conical die shown in Fig. 2, \( D \) and \( d\varepsilon \) are defined by

\[
D = D_0 - 2x \tan \theta
\]

and

\[
d\varepsilon = \frac{dl}{l} = \frac{-2dD}{D} = \frac{4 \tan \theta}{D} dx
\]

where \( x \) is the distance from the entrance to the die zone. Substitution of Eqs. [6] and [7] into Eq. [5] permits the force to be expressed as

\[
F = A_0 K' Q \left[ \frac{1}{D_L} \right]^m - \left( \frac{1}{D_0} \right)^m
\]

where \( Q = (2/3m)(4V_0 D_0^2 \tan \theta)^m \) and \( D_L = D_0 - 2L \tan \theta \).

It can be seen from Eq. [9] that the extrusion force is a sensitive function of \( n \) and \( m \) under conditions of high temperature deformation. Typical values of \( n = 1/m \) are ~2 for superplastic conditions, ~5 for creep conditions, and 5 to 10 under hot working conditions. The coefficient \( K' \) depends not only on the temperature of