The Law of Mixtures Applied to the Plastic Deformation of Two-Phase Alloys of Coarse Microstructures

K. CHO and J. GURLAND

The theory of the mechanical law of mixtures is reviewed and applied to the partition of stress and of strain in two-phase alloys. It is shown that the modified law of mixtures, based on average in situ stress and strain, is valid for small deformations and is compatible with strengthening theories based on continuum and dislocation mechanics. The plastic deformation behavior of two representative two-phase alloys, i.e., a spheroidized high carbon steel and a duplex stainless steel, was studied by taking into account the mutual interactions between phases. The law of mixtures and dispersion strengthening theory were used to evaluate the stress contribution of each phase.

I. INTRODUCTION

A number of technically important alloys consist of mixtures of soft and hard phases or constituents, of coarse microstructures (i.e., with mean-intercept-length of each phase of the order of 1 μm or larger) and compositions such that each constituent is present in relatively large amounts (typically more than 10 pct by volume). Examples of this type of composite alloy are α/β brasses, dual-phase steels, and cemented carbides.

As a group, these alloys offer useful combinations of high strength and good ductility. In plastically deforming composite alloys the softer phase always deforms plastically, but the harder phase may deform either elastically only, or plastically, depending on phase properties, microstructure, and load. In many cases, the plastic deformation behavior of two-phase alloys is characterized by large differences of strain and/or stress between the phases, and the mechanical properties reflect a synergistic interaction of the properties of the constituent phases. The plastic incompatibility between the phases causes both a strong contribution to strain hardening and high local stress concentrations which may lead to premature fracture by void initiation and propagation, unless relieved by local plastic flow. The interaction strain hardening in several two-phase alloys of specific compositions has been successfully estimated by applying the dislocation concepts and continuum mechanics of dispersion hardening theory.

In the present work, we are explicitly concerned with identifying the contribution of each of the phases to the strength and deformation of a two-phase alloy, in general. We will follow a suggestion of Embury and Duncan that the plastic deformation behavior of two-phase alloys can be understood comprehensively by combining the law of mixtures with the dislocation-continuum strain hardening theory. It is an objective of this paper to demonstrate that the law of mixtures for stress and strain, properly modified, provides a valid and useful framework for the analysis of the partition of stress and of strain between the phases during the plastic deformation of two-phase alloys. The analysis will be applied to two representative alloys, namely spheroidized steels (Fe/Fe₃C), the former with elastically deforming hard particles (Fe₃C), the latter with a plastically deforming hard phase (Fe₃C).

II. THE MECHANICAL THEORY OF PHASE MIXTURES

A. Law of Mixtures Applied to Plastic Deformation

The mechanical theory of phase mixtures is well developed for linear properties. For instance, for elastic modulus, Figure 1 shows the high and low limits proposed by Paul, which correspond, respectively, to parallel and series arrangements of the constituents. More closely spaced upper and lower bounds were calculated by Hashin and Shtrikman using geometrical models matching the actual microstructure. For the case of the cemented carbide, WC-Co, represented in the data of Figure 1, they postulated an assembly of composite spheres, each composite sphere consisting of a core of one phase and a spherical shell of the other phase, in the relative volume proportions of the alloys. The good agreement between theory and experiment is indicated in the figure.

In contrast, a comprehensive theory of plastic deformation extending over a range of compositions and mi...
The observed variation of yield strength as a function of composition is given in Figure 2 schematically for different structural arrangements of the constituents, namely (a) discontinuous hard phase, continuous soft phase, (b) continuous hard phase, discontinuous soft phase, (c) both phases continuous, without interaction, and (d) transition between structure (a) and structure (b). The different shapes of the curves are indicative of the important roles of microstructure and phase continuity, respectively, in plastic deformation with ensuing complexity of analysis. Comprehensive reviews of the plasticity of two-phase alloys were published by Fischmeister and Karlsson and Ankem and Margolin.

A number of studies of two-phase alloy systems have considered the plastic properties of the alloys in terms of the states of deformation or flow properties of the constituent phases. The alloy systems so studied include a/β brass, α/β titanium alloys, silver-nickel alloys, ferrite-pearlite carbon steels, ferrite-martensite dual-phase steels, α/γ duplex stainless steels, ferrite-iron carbide spheroidized steels, tungsten carbide-cobalt, and silver-iron, copper-niobium, and copper-iron. Several forms of the law of mixtures have been used to correlate the deformations or properties of the constituents with the properties of the composite alloy. First among these is the classical linear law of mixtures which expresses the properties of a mixture in terms of bulk properties and relative amounts of the constituents. For instance, in Figure 2, line c corresponds to the yield strength of ductile properties of a mixture in terms of bulk properties and relative amounts of the constituents. In this case, the observed linearity, and the fact that the extrapolated end points of the line coincide with the bulk properties of the constituents, demonstrate the invariance of the phase properties over the linear range. Equal strain in both constituents is a necessary requirement for Eq. [1].

However, the yield strengths and flow stresses of most particulate two-phase alloys of technical interest do not follow a linear law of mixtures based on bulk strengths. The properties and stress states of the in situ phases are affected by the microstructure, i.e., size, shape, orientation, and connectivity, and are modified by interaction between the phases. For the purpose of accounting for microstructural and interaction effects, Ankem and Margolin proposed an alteration of the linear law of mixtures, which for yield strength has the following form:

$$\sigma_y = \sigma_{\alpha} V_{\alpha} + \sigma_{\beta} V_{\beta} + I_{\alpha\beta}$$  \[2\]

where $\sigma_{\alpha}$ and $\sigma_{\beta}$ are the bulk yield strengths of α and β phases corrected for variations in grain size and texture, and $I_{\alpha\beta}$ is due to the interaction between α and β phases. The interaction term may be additive or subtractive depending on microstructure, alloy system, and composition and, in principle, its sign and magnitude as a function of composition could account for the shapes of the yield strength curves of Figure 2. The form of Eq. [2] has the advantage of featuring explicitly the interaction term, but it does not directly identify the separate contribution of each phase to the alloy strength.

A more general form of the law of mixtures relating to average stresses and strains in each phase was suggested for empirical use by Tamura, Tomota, and Ozawa and was shown by Fischmeister and Karlsson to describe adequately the distribution of stress and strain between the constituents. This form is called a “modified” law of mixtures because the relevant average stresses and average strains of each phase are in situ values, and not, in general, bulk flow properties, nor necessarily in situ flow properties. For an aggregate or alloy consisting of two phases, α and β, under uniaxial loading, the modified law of mixtures for stress and strains is given by the following equations:

$$\sigma_c = \bar{\sigma}_\alpha V_{\alpha} + \bar{\sigma}_\beta V_{\beta}$$  \[3\]

$$\varepsilon_c = \bar{\varepsilon}_\alpha V_{\alpha} + \bar{\varepsilon}_\beta V_{\beta}$$  \[4\]

where V is the volume fraction, and $\bar{\sigma}$ and $\bar{\varepsilon}$ are the average values, respectively, of the directional components of stress and strain parallel to the direction of the applied load, averaged on planes normal to the load direction (for stress) or along lines parallel to the load direction (for strains). The subscripts α and β refer to phases α and β, respectively, and the subscript c refers to the composite alloy. It is shown in the Appendix that the use of Eqs. [3] and [4] is justified at small strains for both “engineering” stress and strain and “true” stress and strain.

The equations apply to any direction of loading and to all possible structural arrangements of two-phase mixtures, including the limiting cases of parallel and series arrangements of the constituents. In the case of uniaxial loading parallel to the orientation of parallel elements, Eq. [3] gives the appropriate stress values, and Eq. [4] reduces to $V_{\alpha} + V_{\beta} = 1$, at equal stresses in composite and constituents. Similarly, for series loading, Eq. [3] reduces to $V_{\alpha} + V_{\beta} = 1$, at equal stresses in composite and constituents, and Eq. [4] relates the strain of the composite to the average strains of the constituents.