Geometric Models of Internal Shape Change as Shear Bands Form during Plane Strain Extension

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This paper systematically presents geometric models for describing internal shape change during plane strain extension of sheet. The models describe the geometry of plastic flow localization during sheet necking and formation of sample-scale shear bands within the neck. The emphasis is toward identifying three-dimensional, solid element models that correctly describe (1) the heterogeneous initiation of simple shearing flow, inside a band that occupies just a small fraction of the sheet thickness, (2) the gradual growth of bands of simple shearing across the sheet cross section, and (3) the development of a variety of patterns of sample-scale shear bands before sheet fracture. Models are identified that agree with experimental evidence. The geometry of these models provides quantitative links between flow localization throughout a deforming sample and changes in microscopic or crystal-scale flow behavior.

I. INTRODUCTION

Many forming operations require plane strain extension within some regions of the deforming material. Failure can then develop in those regions by the formation of shear bands which collectively cross the sample thickness. Recent experimental studies\(^1,2\) show that sample-scale shear bands develop in a spatially nonuniform manner, at a time while plastic flow localization is taking place throughout the sample. In order to describe this behavior, it is useful to have geometric models for the development of both internal shape change and plastic flow localization. Then experimental results can be compared and interpreted relative to these models. Correct geometrical descriptions of the kinematics of flow are also needed as the starting point for mechanics models of plastic flow localization. Indeed, many of the quantitative aspects of such models arise directly from geometrical relationships that are assumed. While there are a number of kinematic models available in the literature, none fully describes the internal shape changes seen in recent experiments.

This paper considers a number of geometric and kinematic models for describing the shape changes that take place throughout sheet as plane strain extension moves ends of the sheet farther apart. These models are two- and three-dimensional in terms of the shape changes they describe. Each is a simplification of the general approach taken in continuum mechanics to describe sample-scale shape change. The deforming body (in this case, thin sheet) is subdivided into a collection of contiguous solid elements (Figure 1). Each solid element deforms by elastic and plastic flow. External shape change involves sheet extension in one direction, thinning of sheet in a perpendicular direction, and zero shape change in the third, mutually orthogonal direction. Internal shape changes within elements of the sheet can be quite different, but are interconnected by the need for continuity of material between neighboring elements. The models assume that plastic flow localization has already progressed to some extent, and that portions of the sheet are elastically rigid. The models describe shape change in the remainder of the sheet in which flow is still taking place. These models are built on concepts supplied by numerous investigators. An overview of the literature will be provided, after a systematic presentation of several types of kinematical models.

II. PARALLEL SLAB MODELS FOR DESCRIBING NONUNIFORM SHEET EXTENSION AND THROUGH-THICKNESS NECKING

A. Model 1A: No Shearing Flow within Slabs

In this highly simplified model of sheet extension, the sheet is divided into solid slabs, which each lie normal to the direction of imposed sheet extension (Figure 2(a)).
Sheet Thickness

(a) (b) (c)

Fig. 2—Undeformed sheet (a) is divided into slabs which lie normal to the direction of imposed plane strain extension. (b) A model of internal shape change if flow is uniform throughout all slabs. (c) Internal shape change if slabs thin to differing extent and do not remain connected.

Each slab deforms solely by two components of flow: plane strain extension and thinning. Flow is uniform within individual slabs. This model is capable of describing shape change when sheet thins uniformly throughout its volume (Figure 2(b)). However, when there is a gradient of flow along the direction of extension, each slab thins to a different extent, as shown in Figure 2(c). Then the model becomes unrealistic, for it assumes discontinuous changes in the sheet thickness at slab boundaries, which could happen only if slabs were unconnected.

B. Model 1B: Slabs Are Subdivided into Solid Elements. Elements Shear to Maintain Continuity between Slabs.

In this model of shape change, slabs of the previous model (Figure 3(a)) are subdivided so that each slab contains several elements across the sheet thickness (Figure 3(b)). As in model 1A, individual slabs deform with uniform extension and thinning. However, in order to maintain connectivity between deforming elements, wherever there is nonuniform sheet thinning elements undergo an additional component of flow. Elements shear in the plane of the slab toward the nearest sheet surface (Figure 4(a)). The magnitude of through-thickness shearing is shown elsewhere to be:

\[
\gamma = \tan \phi
\]

where \(\gamma\) is engineering shear strain and \(\phi\) is the rotation of element boundaries that lay in the plane of the sheet prior to deformation (Figure 4(b)). This simple shearing flow is an additional but geometrically necessary component of flow that develops whenever there is a gradient of sheet thinning.

The magnitude of \(\phi\) has been measured in experimental studies of ferrite-austenite steel sheet during plane extension. Figure 5(a) shows the cross section of sheet that was

![Sheet Thickness](image)

![Sheet Width](image)

![Plane and Direction of Through-Thickness Shearing](image)

![Continuity between slabs induces through-thickness shearing distortion within solid elements](image)