**A STEADY TWO-DIMENSIONAL CLIMATE MODEL WITH RESIDUAL CIRCULATION**

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**ABSTRACT**

Based on dynamical energy transport and thermodynamic energy balance in the earth's atmosphere-ocean system a steady two-dimensional climate model with residual circulation is proposed. In the model, we include some important physical processes with feedbacks such as ice caps-albedo, water vapor-temperature, etc. The simulated steady temperature field is very close to that of the real atmosphere. The numerical experiments show that doubling of the atmospheric carbon dioxide results in temperature increase of 1~2°C at the low latitude surface and 6~8°C at the high latitude surface. It is shown that a 6% decrease in the solar constant is required for the −10°C ice edge to move from its present latitude ~70° to ~50°.

**I. INTRODUCTION**

Due to the influences of human activities on the atmospheric environment global climatology has become a hot topic of present research in the atmospheric sciences. People have used a number of simple climate models or complex general circulation models to study the effects of some physical processes with feedback mechanisms on the global climatology. Many works showed that doubling the carbon dioxide (CO₂) may cause the global mean temperature to increase 2~3°C, while the increase of the temperature in the high latitude is much greater than in the low latitude (Budyko, 1982; Houghton, 1984). The above results indicate that the contents and distributions of radiative absorbers in the atmosphere (e.g., CO₂, water vapor and ozone), and the north-south transport of heat by atmospheric motions are two important factors in determining the atmospheric temperature distribution. According to this idea we propose a steady two-dimensional climate model which, being not too complicated, includes some important feedback mechanisms. By using appropriate parameterization schemes the computing time is not very large, though many important physical processes in the atmosphere are included in the model.

**II. DYNAMICAL MODEL**


\[
\frac{Du}{Dt} - f*\nu + \frac{\partial \phi}{\partial x} = X, \tag{1}
\]

\[
\frac{Dv}{Dt} + f*\nu + \frac{\partial \phi}{\partial y} = Y, \tag{2}
\]
\[ \frac{\partial \phi}{\partial z} = \frac{R}{H} \theta e^{-\kappa z/H}, \quad (3) \]

\[ \frac{\partial u}{\partial t} + \frac{\partial (\nu \cos \phi)}{\partial y} + \frac{\partial (\rho \omega)}{\rho \partial z} = 0, \quad (4) \]

\[ \frac{D \theta}{Dt} = Q, \quad (5) \]

in which \( z = -H \ln \left( \frac{p}{p_s} \right) \) is log-pressure coordinate,

\[ f^o = f + \frac{u \tan \phi}{a} = 2 \Omega \sin \phi + \frac{u \tan \phi}{a} = \sin \phi \left( 2 \Omega + \frac{u}{a \cos \phi} \right), \quad (6) \]

\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} = \frac{\partial}{\partial t} + \frac{u}{a \cos \phi} \frac{\partial}{\partial \lambda} + \frac{v}{a} \frac{\partial}{\partial \phi} + \frac{w}{a} \frac{\partial}{\partial z}, \quad (7) \]

\[ \theta = Te^{-\kappa z/H} \] is potential temperature, \( H = RT_s/g \) is scale height, \( \kappa = R/c_p = 2/7 \), \( \rho_0 = \rho_s \times e^{-\kappa z/H} \) is the air density, \( X \) and \( Y \) are friction terms and \( Q \) is diabatic heating rate.

Taking zonal average over the equations (1)-(5), we can get the following approximate equations from which our two-dimensional model will be derived:

\[ \frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{u}}{\partial z} = \hat{f} \bar{v} = \bar{X} - \frac{\partial (\bar{u} \bar{v} \cos \phi \bar{\theta})}{\cos \phi \partial y} - \frac{\partial (\rho \bar{u} \bar{w} \bar{\theta})}{\rho \partial z}, \quad (8) \]

\[ \hat{f} \bar{u} + \frac{\partial \bar{\phi}}{\partial y} = 0, \quad (9) \]

\[ \frac{\partial \bar{\phi}}{\partial z} = \frac{R}{H} \theta e^{-\kappa z/H}, \quad (10) \]

\[ \frac{\partial (\bar{v} \cos \phi)}{\cos \phi \partial y} + \frac{\partial (\rho \bar{v} \bar{\theta})}{\rho \partial z} = 0, \quad (11) \]

\[ \frac{\partial \bar{v}}{\partial t} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{v}}{\partial z} = \hat{Q} - \frac{\partial (\bar{v} \bar{v} \cos \phi \bar{\theta})}{\cos \phi \partial y} - \frac{\partial (\rho \bar{w} \bar{\theta} \bar{\theta})}{\rho \partial z}, \quad (12) \]

in which \( \hat{f} = \sin \phi \left( 2 \Omega + \frac{\bar{u}}{a \cos \phi} \right) \). In deriving (9) we have made an assumption that zonal mean flow satisfies the gradient wind balance. The last two terms of the right hand side in (8) and (12) describe the meridional transports of physical quantities by atmospheric eddy motions. Charney and Drazin (1961) proved that the third term on the left and the second term on the right in equation (12) cancel each other for steady, adiabatic and frictionless atmosphere. Based on this point we introduce the following residual meridional circulation:

\[ \bar{v}^* = \bar{v} - \frac{\partial (\rho \bar{u} \bar{\theta})}{\rho \partial z} / \bar{\theta}_z, \quad (13) \]

\[ \bar{w}^* = \bar{w} + \frac{\partial (\bar{u} \bar{v} \cos \phi / \bar{\theta}_z)}{\cos \phi \partial y}. \quad (14) \]

Substituting (13) and (14) into (8)-(12) gives