The Structure and Propagation of Stationary Planetary Wave Packet in the Barotropic Atmosphere

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ABSTRACT

Monthly or seasonally mean anomalies of large-scale atmospheric circulation are better represented by wave packets or their combination. Both qualitative and quantitative analyses of equations of wave packet dynamics, which are obtained by the use of WKB approximation, are very helpful for the understanding of structure, formation and propagation of stationary and quasi-stationary planetary wave packet patterns in the atmosphere. Indeed, these equations of wave packet dynamics can be directly solved by the method of characteristic lines, and the results can be simply and clearly interpreted by physical laws. In this paper, a quasi-geostrophic barotropic model is taken for simplicity, and the wave packets superimposed on several ideal profiles of the basic current and excited by some ideal forcings are investigated in order to make comparison of the accuracy of calculation with the analytical solution. It is revealed that (a) the rays of stationary planetary wave packet do not coincide with but go away from the great circle with significant difference if the shear of the basic zonal flow is not too small; (b) being superimposed on a westerly jet flow with positive shear ($\partial U_z / \partial y > 0$), the stationary wave packets excited by low-latitudinal forcing are first intensified during their northeastward propagation in the Northern Hemisphere, then reach their maximum of amplitude at some critical latitude, and after that weaken again; (c) the connected line of extremes (the positive and negative centres) of wave packet does not coincide with but crosses the ray by an angle, the larger the scale of external forcing, the larger the angle; and (d) the whole pattern of a trapped stationary wave packet is complicated by the interference between the incident and reflected waves.

I. INTRODUCTION

In the monthly mean circulation of the middle and upper troposphere the quasi-stationary disturbances such as troughs, ridges and even closed vortices, superimposed on the basic zonal flow can be considered as compositions of various planetary waves. However, these disturbances—planetary waves take shape of wave packets, because their amplitude and orientation all vary with longitude and latitude. Therefore, to study the structure, formation and the propagation characteristics of these stationary or quasi-stationary wave packets will undoubtedly very helpful for the understanding of monthly mean atmospheric circulations (Zeng et al., 1986; Huang, 1986, 1990).

In this paper we will directly apply the equations for wave packet dynamics to the investigation because the representing disturbances by wave packets is geometrically proper, simple and convenient, and the physical meanings of equations for wave packet dynamics are also very clear.

II. THE MODEL AND THE DYNAMICS EQUATIONS FOR WAVE PACKET

Taking barotropic quasi-geostrophic model in Marcator projection, the
non-dimensional potential vorticity equation linearized with respect to a (non-dimensional) zonal basic flow \( \tilde{u} \) is written as follows (see, Zeng et al., 1986):

\[
\left( \frac{\partial}{\partial t} + \frac{u}{\bar{U}} \frac{\partial}{\partial t} \right) \left[ \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} - \rho^2 \psi' \right] + \tilde{\rho} \frac{\partial \psi'}{\partial x} = Q',
\]

(1)

where \( \psi' \) and \( Q' \) are the perturbations of the stream function and the source intensity (such as heating and dissipation) respectively; \( \tilde{u} = U_x / U^* \sin \theta \) the (nondimensional) angular velocity of the (nondimensional) velocity of the zonal basic flow \( \bar{U}_x, \tilde{\rho} = R_0^{-1} \partial \tilde{q} / \partial y, \)

\( \tilde{q} = 1 + R_0 \left( \beta y - \frac{\partial \tilde{u}}{\partial y} - \rho^2 \tilde{\psi} \right) \)

the (nondimensional) potential vorticity of the basic flow whose stream function is \( \tilde{\psi} \), and \( \rho^2 \) is a function of \( y \) but simplified as a constant here \( (\rho^2 = 1) \); \( x = a \lambda / L^* \) and \( y = (a / L^*) \ln[(1 + \cos \theta) / \sin \theta] \)

are the coordinates along \( \lambda \) (longitude) and \( \frac{\pi}{2} - \theta \) (\( \theta \) the colatitude) respectively; \( U^*, L^* \) and \( T^* = L^* / U^* \) are the characteristic (dimensional) velocity, scale and time respectively; \( a \) is the radius of the Earth; and \( R_0 = U^* / f_0 L^* \) is the Rossby–Kibel' Number (see Zeng et al., 1986, in detail).

The wave packet is represented as follows:

\[
\psi(x,y,t) = \sum_i e^{i \Psi_i(x,y,T)} \exp[i e^{-1} \alpha(x,y,t)],
\]

(2)

where \( e \) is a small parameter, and \( (X,Y) = \varepsilon(x,y) \) and \( T = \varepsilon t \) are the stretched spatial and temporal coordinates respectively.

Substituting (2) into (1), applying WKB method, and denoting

\[
\sigma = -\frac{\partial \alpha}{\partial t}, \quad m = \frac{\partial \alpha}{\partial x}, \quad n = \frac{\partial \alpha}{\partial y},
\]

(3)

we obtain the dispersion relationship

\[
(\sigma - m \tilde{\mu})(m^2 + n^2 + 1) + \tilde{\rho} m = 0,
\]

(4)

and an equation determining the evolution and structure of the zero–order approximation of amplitude \( \Psi_0 \) as follows:

\[
\left( \frac{\partial}{\partial T} + \tilde{C}_g \cdot \nabla \right) A = -A \nabla \cdot \tilde{C}_g + H A(\delta - 2y \tilde{\beta} A),
\]

(5)

where \( \tilde{C}_g = \tilde{T} C_{x \lambda} + \tilde{J} C_{x \gamma} \) is the group velocity whose two components are given by

\[
\begin{aligned}
C_{x \lambda} = \frac{\partial \sigma}{\partial m} = \bar{u} - \bar{\rho} ((m^2 + n^2 + 1) - 2m^2) / (m^2 + n^2 + 1)^2, \\
C_{x \gamma} = \frac{\partial \sigma}{\partial n} = 2mn \bar{\rho} / (m^2 + n^2 + 1)^2;
\end{aligned}
\]

(6)

A is the wave action

\[
A = ![\Psi_0]^2 (m^2 + n^2 + 1)^2 / 2\bar{\rho},
\]

(7)

and the last term on the right hand side of (5) is the source intensity, which consists of dissipation and the release of latent heat of condensation by ascending motion and is here taken as proportional to potential vorticity of the wave packet in a quadratic form with some parameters \( H, \delta \) and \( \gamma, H = 0 \) means that there is a free wave packet.

In the stationary case, \( \sigma = 0 \), we have

\[
(m^2 + n^2 + 1) = \bar{\rho} / \bar{u},
\]

(8)