The Theory Study of the Influence of the Topography on the Cold Frontal Motion

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ABSTRACT

In order to study the characteristics of cold frontal motion over the arbitrary topography, the velocity of cold frontal movement is derived by using the one layer shallow-water model. The results show that there exist the retardation in upwind side and rapid descent in the lee slope when the cold front crosses the topography.

I. INTRODUCTION

Clearly, the movement of front will be significantly influenced by the topography. It has been an interesting research topic by many scientists in recent years. However, it is very difficult to give a complete view or quantitative expression on the topographic effects because the topographic profile is very complex. Davies (1984) has ever made a research for a special orography by a semi-geostrophic shallow-water model and some results have been obtained, but the mathematics treatments used in his paper are only fit for this special orography. For the other arbitrary topography, what are the results of this model? In this paper, a method to compute frontal speed over an arbitrary topography is developed, thus we can understand the contributions of the topography more clearly.

Although the one layer semi-geostrophic shallow-water model used in this paper is very simple, it is still accepted by many scientists. Because the model can provide the basic frontal characteristics relatively well and make the mathematics treatments more easy. What is resulted can also explain the main features of real cold frontal motion.

II. BRIEF DESCRIPTIONS OF THE PHYSICAL MODEL

After Davies (1984), the governing equations for the cold front can be written as:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - f v = - g \frac{\partial (h + \eta)}{\partial x} - \alpha f V_x, \tag{1}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + f u = \alpha f U_x, \tag{2}
\]

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0, \tag{3}
\]

where \((\alpha U_x, \alpha V_x)\) which are constants in time and space are geostrophic velocities in the cold air respectively along the \((x, y)\) direction. \(\alpha\) is defined as \(\frac{\theta - \Delta \theta}{\theta}\). \(\Delta \theta\) is the difference of potential temperature between the warm and cold air. \(\theta\) is the potential temperature of warm air. The other symbols are the same as the ones in Davies (1984).
Let

\[ u = \alpha U_g u^*, \quad v = \alpha V_g + \sqrt{g'\eta} v^*, \quad h = Hh^*, \]

\[ \frac{d}{dt} = \frac{dU_g}{L_f} \frac{d}{dt} \frac{\partial h}{\partial x} = \frac{H}{L_f} \frac{\partial h^*}{\partial x} \frac{\partial \eta}{\partial x} = \frac{\eta_{max}}{L} \frac{\partial \eta^*}{\partial x}, \tag{4} \]

where \( H \) is the depth of the cold air at \( X \to -\infty \), \( \eta_{max} \) is the maximum height and \( L \) is the horizontal scale of the orography respectively. \( L_f = \frac{\sqrt{g'\eta}}{f} \) is the horizontal scale of the front. Thus the nondimensional equations of (1), (2) and (3) can be finally written as

\[ F_r^2 \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) - v = - \frac{\partial h}{\partial x} - \frac{1}{\epsilon H} \frac{\partial \eta}{\partial x}, \tag{5} \]

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} = 1 - u, \tag{6} \]

\[ \frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0, \tag{7} \]

where \( F_r = \frac{\alpha U_g}{\sqrt{g'\eta}} \) is the gravitational Froude number, \( \epsilon = \frac{fL}{\sqrt{g'\eta}} \) is the rotational Froude number and \( \overline{H} = \frac{H}{\eta_{max}} \). So \( \epsilon \overline{H} \) represents the relative magnitude of cold frontal slope and topographic slope.

We might as well make a rough estimation of the aforementioned parameters. For the real cold front, the range of \( \Delta \theta \) is usually from 4K to 10K, and \( H \) is from 4Km to 10Km. If \( \alpha U_g = 10m/s \), then \( g' = 0.13-0.33m/s^2 \), \( F_r^2 = 0.03-0.18 \). Hence semi–geostrophic approximation can be used and (5) can be further simplified as

\[ v = \frac{\partial h}{\partial x} + \frac{1}{\epsilon H} \frac{\partial \eta}{\partial x}, \tag{8} \]

Eqs.(6), (7) and (8) are the basic equations of this physical model. Note that the flow velocity on the downslope can reach the very large values for steep orography and then the order of \( F_r^2 \) cannot be neglected compared with other parameters. If it is the case, the semi–geostrophic approximation will not be valid and too large down–shift velocity will be produced. Thus we will focus our discussion on the situations in which the topographic slope is not too large. At this time the effect of topography is not so strong that the cold front can cross over the topography.

Usually, the movement of the cold front is characterized by that of the ground front line. Thus it is enough to study the velocity of the frontal line.

According to Davies(1984), the following two equations can be obtained from the Eqs. (6)–(8) under the assumption of initial uniform potential vorticity

\[ \frac{\partial^2 (hu)}{\partial x^2} - hu = -1, \tag{9} \]

\[ \frac{\partial^2 h}{\partial x^2} - h + 1 + \frac{1}{\epsilon H} \frac{d^2 \eta}{dx^2} = 0. \tag{10} \]