ON MODULATION OF SOUND BY SOUND

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Frequency shifts of continuous acoustic waves propagating in a medium which arise due to the effect of external acoustic disturbances arbitrarily propagating in the same medium are analyzed. An analytical dependence of the magnitude of the acoustic shift on the angle between the wave vectors of the probing and external waves is obtained. A corresponding directivity pattern is calculated.

It is known that in real media the principle of superposition for acoustic waves is approximate [1]. In [2] we estimated the deviation from this principle for some liquid and gaseous media. Thus, by sending sound or ultrasound waves into a medium, receiving them, and distinguishing the frequency shifts of the received oscillations, one can obtain information about external sources of acoustic signals. An ultrasound beam can act as an acoustic receiving antenna and a microphone. If the ultrasound beam is affected by an acoustic field from some standard source, then valuable information can be obtained about the nonlinear properties of the medium in which the measurements are made, because the nonlinear properties of the medium determine the deviation from the principle of superposition. To analyze these possibilities one should study in more detail the interaction of acoustic waves propagating arbitrarily to each other. In [2], only the simplest case in which the directions of the wave vectors of two acoustic plane waves coincide is considered.

Let continuous plane acoustic waves propagate in the direction of the x axis. For certainty we assume that the emitter is at x = 0, the receiver is at x = L, and the frequency of the emitted waves is constant and equal to \( f_0 \) (probing radiation). Simultaneously, plane acoustic waves with frequency \( F \) (external radiation) propagate in the medium; their direction makes an angle \( \alpha \) with the x axis.

The solution method, as in [2], is as follows. First we find the law of probing wave motion in a medium disturbed by an external acoustic field. It is easily shown that this law is a solution of the differential equation

\[
\frac{dx}{dt} = v_0 + \Delta v_0 \sin \Omega \left( \tau - \frac{x \cos \alpha}{v_0} + t \right)
\]

under the corresponding initial condition. Without loss of generality, the initial condition can be taken in the form

\[ x \big|_{t=0} = 0. \]

Here \( \Omega = 2\pi F \), \( \varphi = \Omega t \) is the initial phase of external acoustic field oscillations at the point \( x = 0 \) at the moment of transmission of probing wave radiation (i.e., at \( t = 0 \)).

The solution of Eq. (1) which satisfies condition (2) has the form

\[
\arctan \left[ \tan \frac{\Omega}{2} \left( \tau - \frac{x \cos \alpha}{v_0} + t \right) - 1 \right] - \arctan \left[ \tan \frac{\Omega}{2} \left( \tau - \frac{x \cos \alpha}{v_0} + t \right) + 1 \right] = \arctan \left[ \tan \frac{\Omega}{2} t - 1 \right] - \arctan \left[ \tan \frac{\Omega}{2} t + 1 \right]
\]


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\[ f = \frac{\Omega t}{2\nu_0} \sqrt{\nu_0^2 (1 - \cos \alpha)^2 - \Delta \nu_0^2 \cos^2 \alpha}, \quad \alpha_0 < \alpha < 2\pi - \alpha_0 \] (3)

and

\[ \ln \left| \frac{\nu_0 (1 - \cos \alpha) \tan \frac{\Omega}{2} \left( t - \frac{x \cos \alpha}{\nu_0} + t \right) - \Delta \nu_0 \cos \alpha - \sqrt{\Delta \nu_0^2 \cos^2 \alpha - \nu_0^2 (1 - \cos \alpha)^2}}{\nu_0 (1 - \cos \alpha) \tan \frac{\Omega}{2} \left( t - \frac{x \cos \alpha}{\nu_0} + t \right) - \Delta \nu_0 \cos \alpha + \sqrt{\Delta \nu_0^2 \cos^2 \alpha - \nu_0^2 (1 - \cos \alpha)^2}} \right| = - \ln \left| \frac{\nu_0 (1 - \cos \alpha) \tan \frac{\Omega}{2} \left( t - \frac{x \cos \alpha}{\nu_0} + t \right) - \Delta \nu_0 \cos \alpha - \sqrt{\Delta \nu_0^2 \cos^2 \alpha - \nu_0^2 (1 - \cos \alpha)^2}}{\nu_0 (1 - \cos \alpha) \tan \frac{\Omega}{2} \left( t - \frac{x \cos \alpha}{\nu_0} + t \right) - \Delta \nu_0 \cos \alpha + \sqrt{\Delta \nu_0^2 \cos^2 \alpha - \nu_0^2 (1 - \cos \alpha)^2}} \right| = \frac{\Omega}{\nu_0} \sqrt{\Delta \nu_0^2 \cos^2 \alpha - \nu_0^2 (1 - \cos \alpha)^2}, \quad -\alpha_0 \leq \alpha \leq \alpha_0, \] (4)

where

\[ A = \sqrt{\left( \frac{\nu_0 (1 - \cos \alpha) + \Delta \nu_0 \cos \alpha}{\nu_0 (1 - \cos \alpha) - \Delta \nu_0 \cos \alpha} \right)}, \quad \alpha_0 = \arccos \frac{\nu_0}{\nu_0 + \Delta \nu_0}. \] (5)

If in expressions (3) or (4) it is assumed that \( x = L \), then from them one can find the time \( \tau = \tau_d \) in which a probing wave reaches the receiver, i.e., the delay time. At constant \( L, \nu_0, \Delta \nu_0, \) and \( \Omega \tau_d \) will be the function of \( \varphi = \Omega t \), i.e., \( t \) can be considered as a current time related to the emitter.

For convenience instead of \( \tau_d \) we introduce the quantity \( \tau_d = L/\nu_0 + \Delta \tau_d \). Then we have instead of (3) and (4)

\[ \arctan \left( A \frac{\tan \frac{\Omega}{2} \left( \Delta \tau_d + \frac{L (1 - \cos \alpha)}{\nu_0} + t \right) - 1}{\tan \frac{\Omega}{2} \left( \Delta \tau_d + \frac{L (1 - \cos \alpha)}{\nu_0} + t \right) + 1} \right) - \arctan \left( A \frac{\tan \frac{\Omega}{2} t - 1}{\tan \frac{\Omega}{2} t + 1} \right) = - \left( \frac{L}{\nu_0} + \Delta \tau_d \right) \frac{\Omega}{2 \nu_0} \sqrt{\nu_0^2 (1 - \cos \alpha)^2 - \Delta \nu_0^2 \cos^2 \alpha} = 0, \quad \alpha_0 < \alpha < 2\pi - \alpha_0 ; \] (6)

\[ \frac{\nu_0 (1 - \cos \alpha) \tan \frac{\Omega}{2} \left( \Delta \tau_d + \frac{L (1 - \cos \alpha)}{\nu_0} + t \right) - \Delta \nu_0 \cos \alpha - \sqrt{\Delta \nu_0^2 \cos^2 \alpha - \nu_0^2 (1 - \cos \alpha)^2}}{\nu_0 (1 - \cos \alpha) \tan \frac{\Omega}{2} \left( \Delta \tau_d + \frac{L (1 - \cos \alpha)}{\nu_0} + t \right) - \Delta \nu_0 \cos \alpha + \sqrt{\Delta \nu_0^2 \cos^2 \alpha - \nu_0^2 (1 - \cos \alpha)^2}} \] \[ - \left( \frac{L}{\nu_0} + \Delta \tau_d \right) \frac{\Omega}{\nu_0} \sqrt{\Delta \nu_0^2 \cos^2 \alpha - \nu_0^2 (1 - \cos \alpha)^2} = 0, \quad -\alpha_0 \leq \alpha \leq \alpha_0. \] (7)