STABLE DOUBLE $LR$ ALGORITHM AND ITS ERROR ANALYSIS

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ABSTRACT. In this paper, the normative matrices and their double $LR$ transformation with origin shifts are defined, and the essential relationship between the double $LR$ transformation of a normative matrix and the $QR$ transformation of the related symmetric tridiagonal matrix is proved. We obtain a stable double $LR$ algorithm for double $LR$ transformation of normative matrices and give the error analysis of our algorithm. The operation number of the stable double $LR$ algorithm for normative matrices is only four sevenths of the rational $QR$ algorithm for real symmetric tridiagonal matrices.

Keywords. normative matrix, double $LR$ transformation, stability.

AMS Subject Classification. 65A15, 65G05.

1. Introduction

The computation of matrix singular values and/or of eigenvalues of Hermite matrices, which is a big branch in numerical algebra, can be reduced to the computation of eigenvalues of real symmetric tridiagonal matrices. For the computation of matrix eigenvalues H. Rutishauser first in 1958 proposed the $LR$ transformation process [1]. As a modification and improvement of the $LR$ transformation process J. G. F. Francis in 1959 proposed the $QR$ transformation process. Today the $QR$ (or $QL$) transformation process has fully, both in theory and in practice, substituted by the $LR$ transformation process [4,5,6] because of its numerical stability.

In this paper we propose a numerical stable algorithm for carrying out the double $LR$ transformation process for normative tridiagonal matrices with origin shifts. The use of stable double $LR$ algorithm instead of the use of rational $QR$ algorithm [2] can save three sevenths of the total operation number in computation of eigenvalues of symmetric tridiagonal matrices.

In Section 2, we define the normative matrices and their double $LR$ transformation with origin shifts. We prove that if $A$ is the normative matrix related to a symmetric
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tridiagonal matrix $S$, $A'$ is the result of double $LR$ transformation of $A$ with origin shift $\sigma$, $S'$ is the result of $QR$ transformation of $S$ with the same origin shift $\sigma$, then $A'$ is the normative matrix related to $S'$. In Section 3, we propose our stable double $LR$ algorithm. In Section 4, we give the error analysis in floating computation of our algorithm. In Section 5, a practical stable double $LR$ algorithm is given.

2. Normative matrices and their double $LR$ transformation

Definition 1. A real tridiagonal matrix $A$ is said to be a normative matrix, if its super-diagonals are units and its subdiagonals are all nonnegative:

$$
A = \begin{pmatrix}
  a_1 & 1 & & & \\
  b_2 & a_2 & 1 & & \\
  & * & * & * & \\
  & & * & * & * \\
  & & & * & * \\
  b_{n-1} & a_{n-1} & 1 & & \\
  b_n & a_n & & & 
\end{pmatrix}
$$

$$
b_i > 0, \quad i = 2, \ldots, n. \quad (1)
$$

We say the normative matrix $A$ in Eq. (1) and a symmetric tridiagonal matrix

$$
S = \begin{pmatrix}
  a_1 & \beta_2 & & & \\
  \beta_2 & a_2 & \beta_3 & & \\
  & * & * & * & \\
  & & * & * & * \\
  & & & * & * \\
  \beta_{n-1} & a_{n-1} & \beta_n & & \\
  \beta_n & a_n & & & 
\end{pmatrix}
$$

$$
\beta_i > 0, \quad i = 2, \ldots, n. \quad (2)
$$

are related (to each other), if $b_k = \beta_k^2$, $k = 2, \ldots, n$.

When $A$ in Eq.(1) and $S$ in Eq.(2) are related, they have the same spectral set, and moreover, they are diagonally similar if they are irreducible.

Definition 2. The double $LR$ transformation of a normative matrix $A$ with origin shift $\sigma$ can be shown in the following process (if it is possible):

$$
A - \sigma I =: LR, \quad \text{(decomposition)}, \quad (3.1)
$$

$$
\hat{A} := RL, \quad \text{(recomposition)}, \quad (3.2)
$$

$$
\hat{A} =: \hat{L}\hat{R}, \quad \text{(decomposition)}, \quad (3.3)
$$

$$
A' := \hat{R}\hat{L} + \sigma I, \quad \text{(recomposition)}. \quad (3.4)
$$

Here the matrix decompositions are Doolittle decompositions. It is trivial that the super-diagonals of $R$, $\hat{R}$, $A$ and $A'$ are units.

The following theorem is important for us.

Theorem 1. Suppose normative matrix $A$ and symmetric tridiagonal matrix $S$ are related, $S'$ is the result of $QR$ transformation of $S$ with origin shift $\sigma$. If $A-\sigma I$ is