ASYMPTOTIC BEHAVIOR FOR A CLASS OF ELLIPTIC EQUIVALUED SURFACE BOUNDARY VALUE PROBLEM WITH DISCONTINUOUS INTERFACE CONDITIONS

Cai Zhijie

Abstract. Spontaneous potential well-logging is one of the important techniques in petroleum exploitation. A spontaneous potential satisfies an elliptic equivalued surface boundary value problem with discontinuous interface conditions. In practice, the measuring electrode is so small that we can simplify the corresponding equivalued surface to a point. In this paper, we give a positive answer to this approximation process: when the equivalued surface shrinks to a point, the solution of the original equivalued surface boundary value problem converges to the solution of the corresponding limit boundary value problem.

1. Introduction

Spontaneous potential well-logging is a common and important method in petroleum exploitation (cf. [3]). In order to make the corresponding log interpretation chart, it is usually supposed that the geometrical structure of the formation, the resistivity in each subdomain and the spontaneous potential difference on each interface are all known. Then the spontaneous potential satisfies an elliptic equivalued surface boundary value problem with jump conditions on interfaces. As a usual convention in petroleum geophysics, suppose that the formation is symmetric about the well axis and the central plane (cf. Figure 1), we consider the problem in the domain \( \{(r, z) \mid 0 \leq r \leq \sqrt{x^2 + y^2} \leq R, 0 \leq z \leq Z\} \) on the \((r, z)\) plane, where \(R\) and \(Z\) are suitably large positive constants. In this case, the spontaneous potential \( u = u(r, z) \) satisfies the following equation and boundary conditions in each subdomain \( \Omega_i \) (\(i = 1, \ldots, 4\))

\[
Lu = 0 \quad \text{in} \ \Omega_i \ (1 \leq i \leq 4),
\]

\[
u = E_5(C) \quad \text{in} \ \Gamma_{11},
\]
\[ u = 0 \quad \text{in } \Gamma_{12}, \]
\[ \frac{r}{Re} \frac{\partial u}{\partial n} = 0 \quad \text{in } \Gamma_{2j} \quad (1 \leq j \leq 7), \]
\[ u = \text{constant (unknown)} \quad \text{in } \Gamma_0, \]
\[ \int_{\Gamma_0} \frac{r}{Re} \frac{\partial u}{\partial n} ds = 0, \]
\[ u^+ - u^- = E_j(s) \quad \text{in } \gamma_j \quad (1 \leq j \leq 5), \]
\[ \left( \frac{r}{Re} \frac{\partial u}{\partial n} \right)^+ = \left( \frac{r}{Re} \frac{\partial u}{\partial n} \right)^- \quad \text{in } \gamma_j \quad (1 \leq j \leq 5), \]

where

\[ L = \frac{\partial}{\partial r} \left( \frac{r}{Re} \frac{\partial}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{r}{Re} \frac{\partial}{\partial z} \right) \]

is a quasiharmonic operator and \( Re \) is the resistivity of the layer. As usual, suppose